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On the exact solution for peristaltic flow of couple-stress fluid with wall properties

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This communication reports the peristaltic motion of an electrically conducting couple-stress fluid in a channel with complaint walls. Mathematical model subjected to long wavelength and low Reynolds number approximations is presented. A closed form exact solution for the dimensionless stream function is derived. Behaviors of different physical parameters including Hartman number (M), couple-stress fluid parameter (γ), elastic parameters (E_1, E_2, E_3, E_4, E_5) and amplitude ratio (ε) are thoroughly examined through the graphical results for velocity and stream function. The study reveals that fluid velocity significantly increases with an increase in the couple-stress fluid parameter and decreases with an increase in the magnetic field strength.

Keywords: Couple-stress fluid, compliant wall, peristalsis, magnetic field, analytic solution.

INTRODUCTION

Peristaltic motion is the form of fluid transport that occurs due to the waves travelling along the walls of a channel/tube. This mechanism appears in many industrial and physiological processes including rollers and finger pumps to pump sanitary and corrosive fluids. The peristaltic activity is quite prevalent in the gastrointestinal tract for food bolus transportation. It also occurs in the urinary tract to transport urine from kidney to bladder, in small blood vessels to transport blood, spermatozoa transport in the *ductus efferentes*, etc. Haroun [1] studied the peristaltic flow of third grade fluid in an asymmetric channel. Peristaltic flow of fourth grade fluid in an inclined channel is also discussed by Haroun [2]. Kothandapani and Srinivas [3] presented the peristaltic flow of Newtonian fluid in an inclined asymmetric channel. Here, the fluid saturates the porous medium. Kothandapani and Srinivas [4] discussed the effect of magnetic field on the peristaltic flow of Jeffrey fluid in an asymmetric channel. Ebaid [5] studied the effects of magnetic field and wall slip conditions on the peristaltic transport of Newtonian fluid in an asymmetric channel. Muthuraj and Srinivas [6] presented the mixed convective heat and mass transfer effects on the peristaltic flow in a vertical wavy channel with porous medium. Srinivas *et al.* [7] extended the work of Muthuraj and Srinivas [6] for an asymmetric channel. Srinivas and Muthuraj [8] discussed the effects of chemical reaction and space porosity on MHD mixed convective peristaltic flow in a vertical asymmetric channel. Heat transfer in the pulsatile flow through a vertical annulus was discussed by Elmaboud and Mekheimer [9]. Peristaltic flow of an electrically conducting micropolar fluid was described by Mekheimer [10].

It is now recognized that fluids involved in various industrial and physiological processes are non-Newtonian. In view of flow diversity in nature, all non-Newtonian fluids cannot be described by a single constitutive relationship between stress and shear rate. Due to this reason several constitutive equations describing the motions of such fluids have been proposed by the researchers. The associated equations are mathematically more complex and higher-ordered than the governing equations for a Navier-Stokes fluid. Couple-stress fluid is one of the non-Newtonian fluids that describe rheologically complex fluids such as liquid crystals, polymeric suspensions, infected urine, animal and human blood and many lubricants. Recently, many researchers discussed the peristaltic flow of couple- stress fluid. Mekheimer [11] analyzed the effect of induced magnetic field on the peristaltic flow of couple-stress fluid. Mekheimer and Elmaboud [12] studied the peristaltic flow of couple-stress fluid in an annulus. Nadeem and Akram [13] extended the work of

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Mekheimer [11] for an asymmetric channel. Rao and Rao [14] discussed the influence of heat transfer on the peristaltic transport of couple-stress fluid through a porous medium. Eldabe *et al.* [15] studied the (1)MHD peristaltic flow of couple-stress fluid with heat and mass transfer through a porous medium. Heat transfer analysis in the peristaltic flow of couple-stress fluid through asymmetric channel is performed by Elmaboud *et al*. [16].

In all above mentioned studies, the effect of wall properties is not taken into account. The consideration of wall properties of the channel in peristaltic motion is realistic. Due to this reason the peristaltic flow in a channel with wall properties has great importance in the natural processes that exist in industry and physiology. Muthu *et al*. [17] discussed the effect of wall properties on the peristaltic motion of micropolar fluid. Radhakrishnamacharya and Srinivasulu [18] studied the influence of wall properties on the peristaltic motion with heat transfer. Kumari and Radhakrishnamacharya [19] analyzed the effect of slip boundary condition on the peristaltic transport in an inclined channel with wall properties. Kothandapani and Srinivas [20] discussed the influence of wall properties on the MHD peristaltic flow with heat transfer and porous medium. Srinivas and Kothandapani [21] presented the heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls. Hayat *et al.* [22] extended the work of Srinivas and Kothandapani [18] for the flow of second grade fluid. Hayat *et al.* [23] discussed the peristaltic flow of Maxwell fluid in an asymmetric channel with wall properties. Mustafa *et al.* [24] analyzed the effect of wall properties on the peristaltic transport of nanofluid. Mustafa *et al.* [25] extended their work [24] by considering the slip boundary conditions. Hina *et al*. [26] studied the peristaltic flow of pseudoplastic fluid in a curved channel with wall properties.

The objective here is to examine the magnetohydrodynamic (MHD) peristaltic flow of couple-stress fluid with wall properties. To our knowledge, no such attempt is made yet. Problem formulation is presented. Closed form exact solutions for velocity and stream function are constructed. Effects of couple-stress parameter, Hartman number and compliant wall parameters are also discussed in detail.

FORMULATION OF THE PROBLEM

Consider the peristaltic flow of couple-stress fluid in a compliant walls channel. The channel width is taken as $2d_1$. The $x-$ and $y-$ axes are along and perpendicular to the channel walls, respectively. An incompressible and electrically conducting fluid is considered. A uniform magnetic field of strength B_0 is applied in the $y-$ direction. The flow is induced due to the sinusoidal waves on the compliant walls of the channel and the wave shapes are described as follows:

$$y = \pm\eta(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(x - ct)\right]. \quad (1)$$

In the above equation c is the wave speed, a the wave amplitude and λ the wavelength. The governing equations for the problem under consideration are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$\rho\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]u = -\frac{\partial p}{\partial x} + \mu\nabla^2 u - \eta\nabla^4 u - \sigma B_0^2 u, \quad (3)$$

$$\rho\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]v = -\frac{\partial p}{\partial y} + \mu\nabla^2 v - \eta\nabla^4 v, \quad (4)$$

and the boundary conditions are

$$u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = \pm\eta, \quad (5)$$

$$\left[-\tau\frac{\partial^3}{\partial x^3} + m\frac{\partial^3}{\partial x\partial t^2} + d\frac{\partial^2}{\partial t\partial x} + B\frac{\partial^5}{\partial x^5} + H\frac{\partial}{\partial x}\right]\eta = \mu\nabla^2 u - \eta_1\nabla^4 u - \sigma B_0^2 u - \rho\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)u \text{ at } y = \pm\eta. \quad (6)$$

In the above equations u and v denote the velocities in the $x-$ and $y-$ directions, respectively, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, p denotes the pressure, σ is the electrical conductivity of the fluid, τ the elastic tension in the membrane, m the mass per unit area, d the coefficient of viscous damping, B the flexural rigidity of the plate, H the spring stiffness, η_1 the couple-stress fluid parameter, μ the dynamic viscosity and ρ the density.

We define the following dimensionless variables

$$u^* = \frac{u}{c},\ v^* = \frac{v}{c},\ x^* = \frac{x}{\lambda},\ y^* = \frac{y}{d_1},\ t^* = \frac{ct}{\lambda},$$
$$\gamma = d_1\sqrt{\frac{\mu}{\eta_1}},\ p^* = \frac{d_1^2 p}{c\lambda\mu},\ \eta^* = \frac{\eta}{d_1}. \qquad (7)$$

The problem statement in the dimensionless variables is given by

$$\mathrm{Re}\left[\delta\frac{\partial}{\partial t}+u\delta\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right]u = -\frac{\partial p}{\partial x}+\nabla^2 u-\frac{1}{\gamma^2}\nabla^4 u - M^2 u, \qquad (8)$$

$$\mathrm{Re}\,\delta\left[\delta\frac{\partial}{\partial t}+u\delta\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right]v = -\frac{\partial p}{\partial y}+\nabla^2 v-\frac{1}{\gamma^2}\nabla^4 v, \qquad (9)$$

$$u=0,\quad \frac{\partial^2 u}{\partial y^2}=0 \text{ at } y=\pm\eta=\pm\left(1+\varepsilon\sin 2\pi(x-ct)\right), \qquad (10)$$

$$\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}+E_4\frac{\partial^5}{\partial x^5}+E_5\frac{\partial}{\partial x}\right]\eta$$
$$=\nabla^2 u-\frac{1}{\gamma^2}\nabla^4 u - M^2 u \qquad (11)$$
$$-\mathrm{Re}\left(\delta\frac{\partial}{\partial t}+u\delta\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u \quad \text{at } y=\pm\eta.$$

In the above equations $\nabla^2=\delta^2\partial^2/\partial x^2+\partial^2/\partial y^2$, ε $(=a/d_1)$ is the amplitude ratio, δ $(=d_1/\lambda)$ the wave number, Re $(=cd_1/\upsilon)$ the Reynolds number, $M\left(=B_0 d_1\sqrt{\sigma/\mu}\right)$ the Hartman number, γ the couple-stress fluid parameter and $E_1(=-\tau d_1^3/\lambda^3\mu c)$, $E_2(=mcd_1^3/\lambda^3\mu)$, $E_3(=dd_1^3/\lambda^2\mu)$, $E_5(=Hd_1^3/\lambda c\mu)$ are the non-dimensional elasticity parameters.

Denoting the stream function $\psi(x,y,t)$ by

$$u=\frac{\partial\psi}{\partial y},\ v=-\delta\frac{\partial\psi}{\partial x}, \qquad (12)$$

we can see that the equation of continuity (2) is automatically satisfied. Introducing stream function in the flow problem and then applying long wavelength and low Reynolds number approximations, we obtain

$$\frac{\partial^3\psi}{\partial y^3}-\frac{1}{\gamma^2}\frac{\partial^5\psi}{\partial y^5}-M^2\frac{\partial\psi}{\partial y}=\frac{\partial p}{\partial x}, \qquad (13)$$

$$\frac{\partial p}{\partial y}=0, \qquad (14)$$

with boundary conditions

$$\frac{\partial\psi}{\partial y}=0,\qquad \frac{\partial^3\psi}{\partial y^3}=0 \qquad \text{at } y=\pm\eta, \qquad (15)$$

$$-\gamma^2\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}+E_4\frac{\partial^5}{\partial x^5}+\right. \quad (7)$$
$$=\frac{\partial^5\psi}{\partial y^5}-\gamma^2\frac{\partial^3\psi}{\partial y^3}+M^2\gamma^2\frac{\partial\psi}{\partial y} \quad \text{at} \quad (y=\pm\eta). \qquad (16)$$

Equations (13) and (14) yield

$$\frac{\partial^6\psi}{\partial y^6}-\gamma^2\frac{\partial^4\psi}{\partial y^4}+M^2\gamma^2\frac{\partial^2\psi}{\partial y^2}=0. \qquad (17)$$

Solving equation (17) subject to the boundary conditions (15) and (16), we have

$$\psi=C_1\sinh m_1 y+C_2\sinh m_2 y+Ly, \qquad (18)$$

$$u=m_1C_1\cosh m_1 y+m_2C_2\cosh m_2 y+L, \qquad (19)$$

where

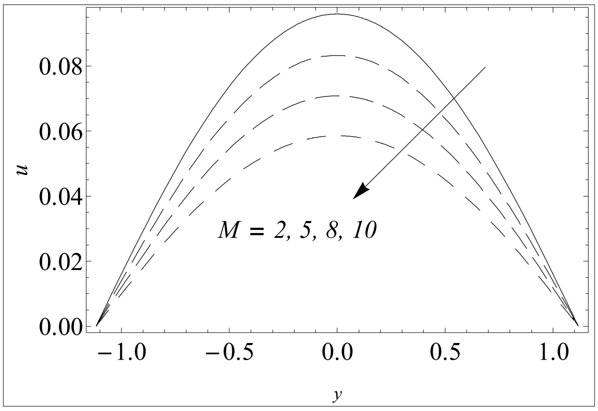
$$m_1=\sqrt{\frac{\gamma^2+\sqrt{\gamma^4-4M^2\gamma^2}}{2}},\qquad m_2=\sqrt{\frac{\gamma^2-\sqrt{\gamma^4-4M^2\gamma^2}}{2}},$$

$$C_1=\frac{m_2^2L}{m_1\left(m_1^2-m_2^2\right)\cosh m_1\eta},\qquad C_2=\frac{-m_1^2L}{m_2\left(m_1^2-m_2^2\right)\cosh m_2\eta},$$

$$L=\frac{8\pi^3\varepsilon}{M^2}\left\{\left(E_1+E_2-4\pi^2E_4-\frac{E_5}{4\pi^2}\right)\cos 2\pi(x-t)-\frac{E_3}{2\pi}\sin 2\pi(x-t)\right\}.$$

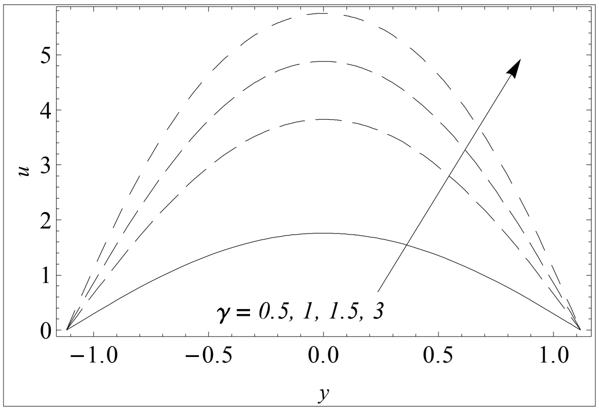
RESULTS AND DISCUSSION

This section discusses the variations of different emerging parameters on the axial velocity u and stream function ψ. The variation of Hartman number (M), amplitude ratio (ε), elastic tension in the membrane (E_1), mass per unit area (E_2), coefficient of viscous damping (E_3), flexural rigidity of the plate (E_4), spring stiffness (E_5) and couple-stress parameter (γ) are examined. Fig. 1 elucidates the behavior of different parameters involved in the axial velocity. Fig. 1a shows that velocity decreases with an increase in Hartman number M.



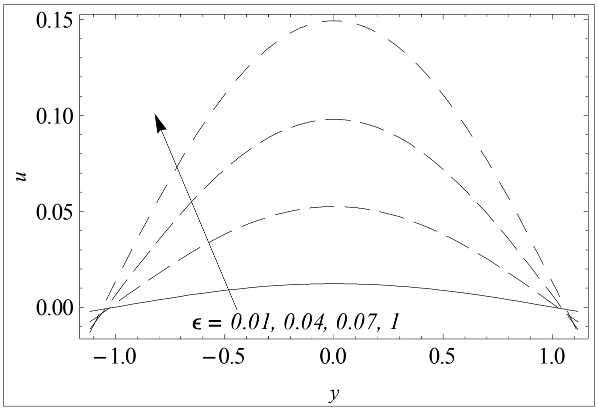
(1a)

Fig. 1(a). Variation of M on u when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $\gamma = 0.1$; $x = 0.2$; $t = 0.1$.



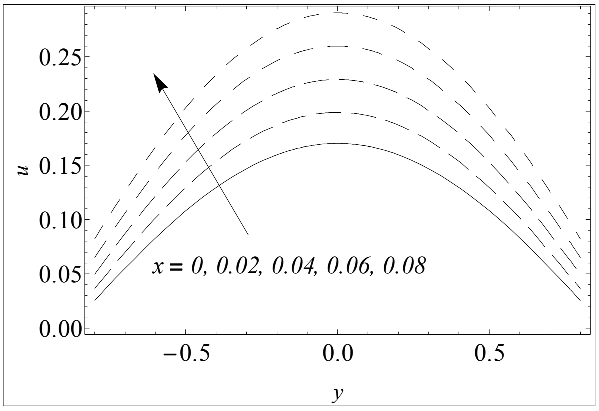
(1b)

Fig. 1(b). Variation of γ on u when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $M = 2$; $x = 0.2$; $t = 0.1$.



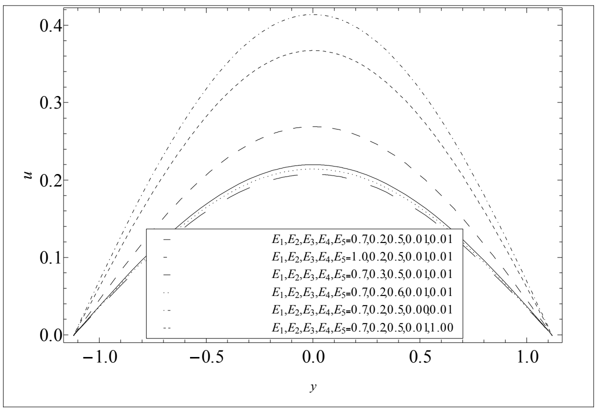
(1c)

Fig. 1(c). Variation of ε when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\gamma = 0.2$; $M = 2$; $x = 0.2$ $t = 0.1$.



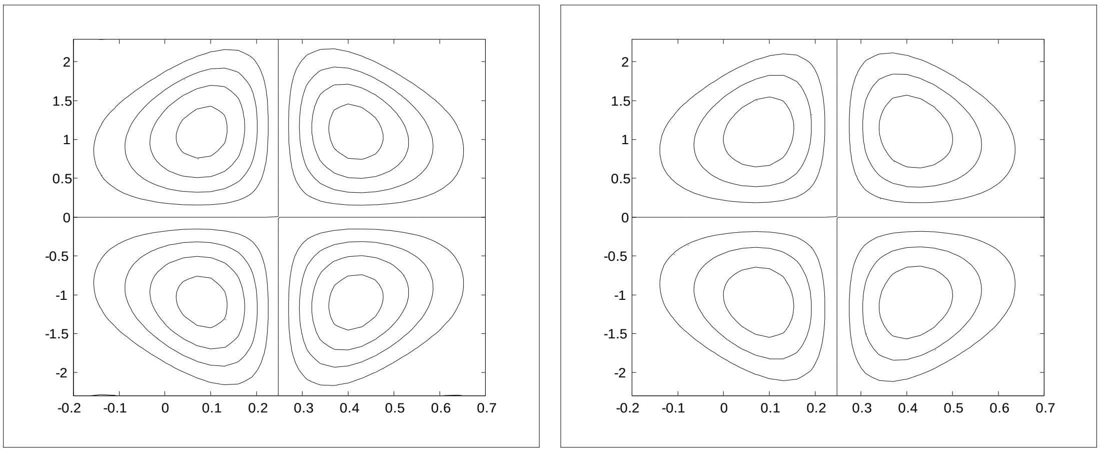
(1d)

Fig. 1(d). Variation of x when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $\gamma = 0.2$; $M = 2$; $t = 0.1$.



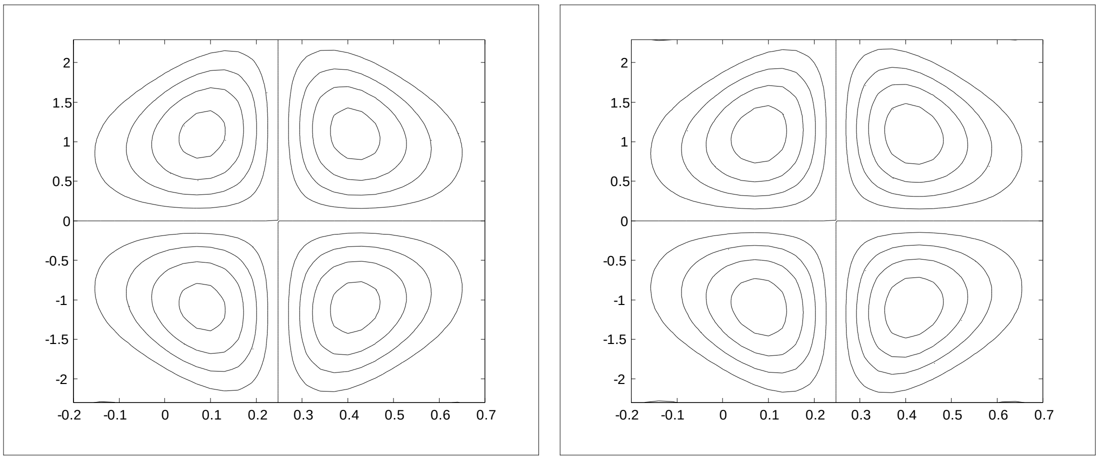
(1e)

Fig. 1(e). Variation of compliant wall parameters when $\varepsilon = 0.2$; $\gamma = 0.2$; $M = 2$; $x = 0.2$; $t = 0.1$.



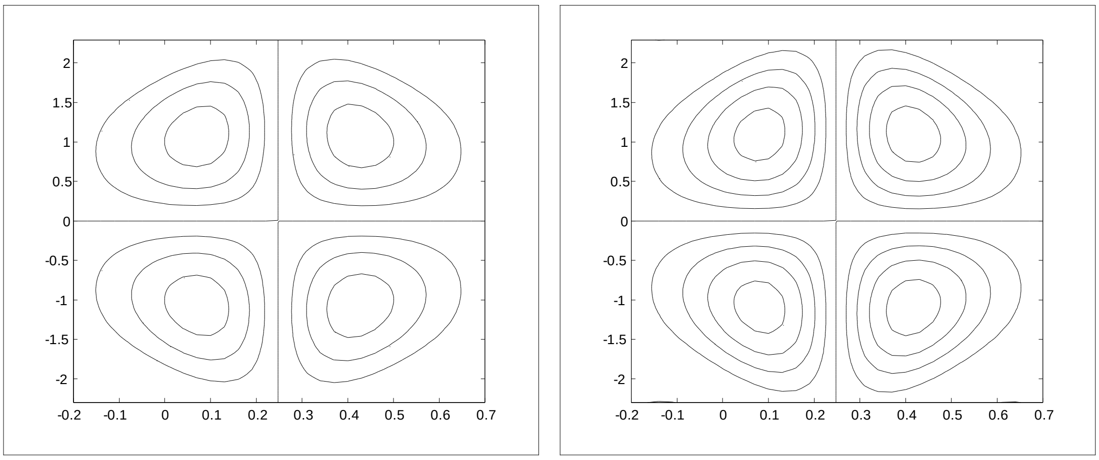
(2a) (2b)

Fig. 2. Streamlines for $E_1 = 1$; $E_2 = 0.5$; $E_3 = 0.1$; $E_4 = 0.01$; $E_5 = 1$; $\varepsilon = 0.18$; $t = 0$; $\gamma = 0.02$; (a): $M = 2$; (b): $M = 2.2$.



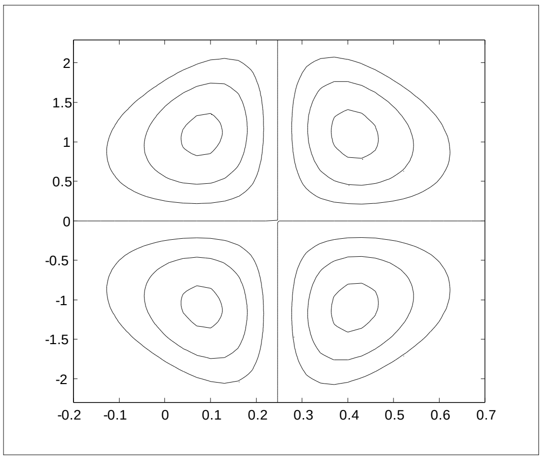
(3a) (3b)

Fig. 3. Streamlines for $E_1 = 1$; $E_2 = 0.5$; $E_3 = 0.1$; $E_4 = 0.01$; $E_5 = 1$; $\varepsilon = 0.18$; $t = 0$; $M = 2$; (a): $\gamma = 0.01$; (b): $\gamma = 0.03$.

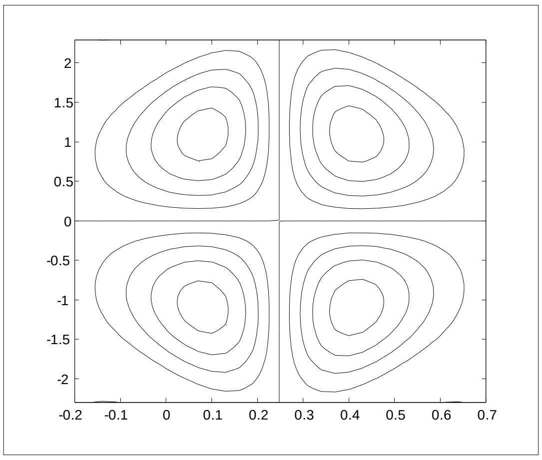


(4a) (4b)

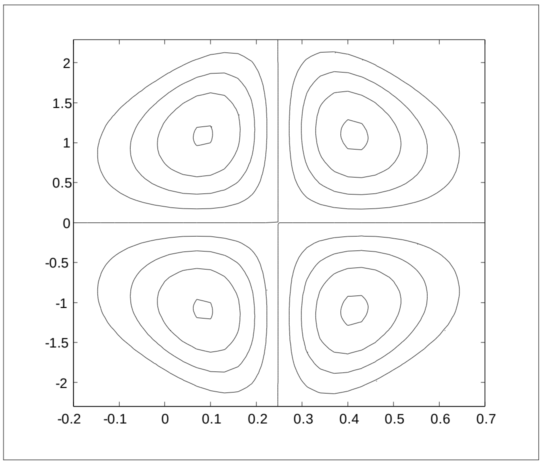
Fig. 4. Streamlines for $E_1 = 1$; $E_2 = 0.5$; $E_3 = 0.1$; $E_4 = 0.01$; $E_5 = 1$; $\gamma = 0.02$; $t = 0$; $M = 2$; (a): $\varepsilon = 0.15$; (b): $\varepsilon = 0.18$.



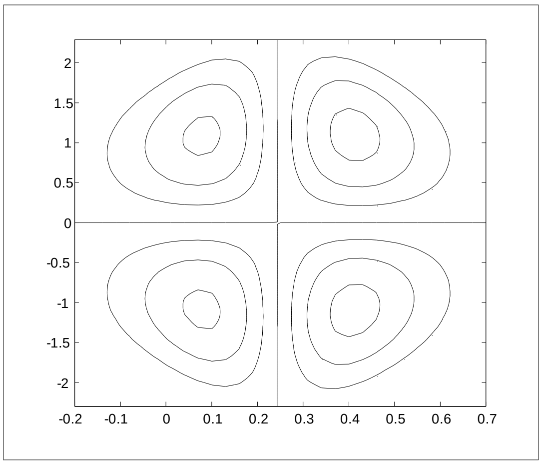
(5a)



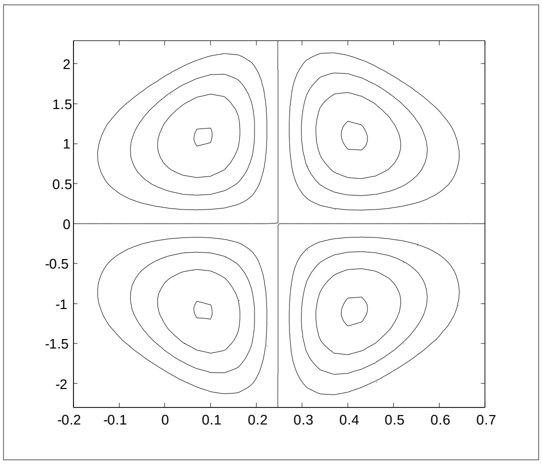
(5b)



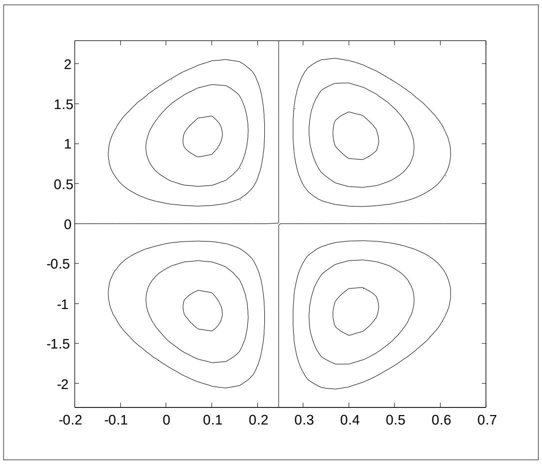
(5c)



(5d)



(5e)



(5f)

Fig. 5. Streamlines for $\varepsilon = 0.18$; $M = 2$; $\gamma = 0.02$; $t = 0$; (a): $E_1 = 0.7$; $E_2 = 0.5$; $E_3 = 0.1$; $E_4 = 0.01$; $E_5 = 1$; (b): $E_1 = 1$; $E_2 = 0.5$; $E_3 = 0.1$; $E_4 = 0.01$; $E_5 = 1$; (c): $E_1 = 0.7$; $E_2 = 0.7$; $E_3 = 0.1$; $E_4 = 0.01$; $E_5 = 1$; (d): $E_1 = 0.7$; $E_2 = 0.5$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; (e): $E_1 = 0.7$; $E_2 = 0.5$; $E_3 = 0.1$; $E_4 = 0.005$; $E_5 = 1$; (f): $E_1 = 0.7$; $E_2 = 0.5$; $E_3 = 0.1$; $E_4 = 0.01$; $E_5 = 1.2$.

Hartman number is the ratio of magnetic force to viscous force. Magnetic force being transverse to the flow direction, causes resistance to the flow and hence slows down the fluid motion. Fig. 1b depicts that velocity increases by increasing the couple-stress fluid parameter. Fig. 1c analyzes the behavior of amplitude ratio ε on axial velocity. It is observed that by increasing the amplitude ratio the axial velocity increases. This is due to the fact that increasing values of the amplitude ratio ε correspond to an increase in the amplitude of the wave across the channel which thereby increases the fluid velocity within the channel. Fig. 1d is sketched to perceive the behavior of velocity distribution at different locations in the channel. It is seen that the axial velocity increases along the positive $x-$ direction. The influences of compliant wall parameters (E_1 , E_2 , E_3 , E_4 and E_5) on the velocity can be captured from Fig.1e. It is observed that the velocity distribution increases with an increase in E_1 and E_2, whereas it decreases when E_3, E_4 and E_5 are increased. These observations may be inferred to the fact that increase in the elasticity of the channel walls assists the flow whereas damping and stiffness of the wall resists the flow.

Figs. 2-5 illustrate the behaviour of embedded parameters on the stream function

Generally, the shape of streamlines is analogous to the wave travelling along the walls of the channel. Under certain conditions these streamlines split and enclose a bolus which moves along with the wave across the channel. The circulation and size of the trapped bolus are presented in these Figs. Fig. 2 shows the impact of Hartman number or equivalently the magnetic field on the streamlines. It is observed that the size of trapped bolus and the number of circulations decrease with an increase in M . Fig. 3 shows that the size of bolus increases with an increase in couple-stress fluid parameter γ. Fig. 4 analyzes the effect of amplitude ratio ε on the stream function. It is clear that size of the trapped bolus and number of circulations are increased by increasing the amplitude ratio ε . Fig. 5 is plotted to examine the behavior of the wall elasticity parameters. It is observed that the size of trapped bolus increases by increasing E_1, E_2 and it decreases by increasing E_3 , E_4 and E_5.

CONCLUSIONS

In this article, MHD peristaltic motion of couple-stress fluid is discussed in a compliant walls channel. A closed form exact solution of the problem is presented after applying long wavelength and low Reynolds number assumptions. The effects of different parameters on the axial velocity and streamlines are thoroughly discussed. It is observed that the axial velocity and the size of trapped bolus decreases by increasing the Hartman number, wall damping parameters and wall stiffness parameters, whereas axial velocity and size of trapped bolus increases by increasing the couple-stress fluid parameter, amplitude ratio and wall elasticity parameters. The present study finds an important application in describing blood flow in arteries since the rheological properties of blood can be adequately described through the couple-stress fluid model. Moreover, the human arteries and veins possess the properties of elasticity, damping and stiffness which are also taken into consideration in this work.

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ОТНОСНО ТОЧНОТО РЕШЕНИЕ ЗА ПЕРИСТАЛТИЧНО ТЕЧЕНИЕ НА ФЛУИД СЪС СПРЕГНАТИ НАПРЕЖЕНИЯ И ПРОМЕНЛИВИ СВОЙСТВА НА СТЕНАТА

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(Резюме)

В тази работа се разглежда перисталтичното движение на електропроводящ флуид със спрегнати напрежения (couple-stress fluid) в канал със стени с променлива твърдост. Въведен е математичен модел за ламинарно вълново течение при голяма дължина на вълната. Получено е точно решение за безизмерната токова функция. Подробно са изследвани различни параметри, като числото на Hartman number (M), параметъра на спрегнати напрежения (γ), параметрите на еластичността (E1; E2; E3; E4; E5) и отношението на амплитудите (ε) чрез резултатите за скоростта на течението и токовата функция. Изследването показва, че скоростта на течението се повишава значително с нарастване на параметъра на спрегнатите напрежения и се понижава, когато се приложи магнитно поле.