Phase recovery from fringe patterns with global carriers: approach based on Hilbert transform and wavelet de-noising techniques

P. Sharlandjiev*, N. Berberova, D. Nazarova and G. Stoilov

Institute for Optical Materials and Technologies “Acad. J. Malinowski”, Bulgarian Academy of Sciences,
“Acad. G. Bonchev” str. Bl. 109, 1113 Sofia, Bulgaria
Institute of Mechanics, Bulgarian Academy of Sciences,
„Akad. G. Bonchev“ Str., block 4, Sofia PS-1113, Bulgaria,
Received October 10, 2016; Revised November 10, 2016

Fringe pattern analysis is an important task in optical metrology. Fringe patterns can be formed by optical interference, by projection techniques, by overlapping two similar wave structures, etc. Patterns with constant global angular carriers represent straight lines in the field-of-view. The presence of an object under investigation distorts the fringes. Analysis of these distortions is called also phase recovery and it is widely used in many applications of science and engineering, i.e. for retrieval of surface topography of 3D objects. Usually, three steps are discerned: pre-processing (noise reduction, background zeroing), phase retrieval (extraction of the phase distribution), and post-processing (unwrapping, smoothing and ‘cleaning’).

Herein we present an approach based on a combination of Hilbert transform for phase recovery and different wavelet techniques for de-noising and smoothing. Different types of noise are considered: Gaussian additive noise, multiplicative intensity dependent (speckle) noise, high frequency environmental noise, jitter, fringe distortion due to non-sinusoidal modulation (presence of second and third harmonics in the fringes). Each type of noise can be considered separately or all of them – cumulatively.

Keywords: Optical metrology; Fringe projection; Numeric filter techniques

INTRODUCTION

Fringe pattern analysis is an important task in optical metrology. Fringe patterns can be formed by optical interference, by projection techniques, by overlapping two similar wave structures, etc. [1]. Patterns with constant global angular carriers represent straight lines in the field-of-view. The presence of an object under investigation distorts the fringes. Analysis of these distortions is called also phase recovery and it is widely used in many applications of science and engineering, i.e. for retrieval of surface topography of 3D objects [1,2]. Usually, three steps are discerned: pre-processing (noise reduction, background zeroing), phase retrieval (extraction of the phase distribution), and post-processing (unwrapping, smoothing and ‘cleaning’).

Herein we present an approach based on a combination of Hilbert transform [2] for phase recovery and different wavelet techniques for de-noising and smoothing. Different types of noise are considered: Gaussian additive noise, multiplicative intensity dependent (speckle) noise, high frequency environmental noise, jitter, fringe distortion due to non-sinusoidal modulation (presence of second and third harmonics in the fringes). Each type of noise can be considered separately or all of them – cumulatively.

The effectiveness of the Hilbert transform is compared to that of complex Gabor transform [1], which can be used for phase retrieval, too. Wavelet de-noising is compared to that of windowed Fourier filter [2] and others filters, as well (see below).

In all simulations the phase unwrapping is done by the Itoh approach [3]. The smoothed final result is ‘cleared’ by an adaptive Wiener filter [1].

PHASE AND NOISE MODELS; FRINGES AND COMPUTATIONAL PROCEDURES

The phase model (pm) in the simulations below is the function “peaks”, which is more or less accepted as a standard in surface profile analysis:

* To whom all correspondence should be sent:
E-mail: pete@iomt.bas.bg
\[ p = 3 \cdot (1 - x)^2 \cdot \exp(-(x)^2 - (y + 1)^2) - \\
1/3 \cdot \exp(-(x + 1)^2 - (y)^2) - \\
10 \cdot (x/5 - x^3 - y^3) \cdot \exp(-x^2 - y^2); \]

\[ pm(x, y) = 3.5 \cdot p; \quad 0 \leq x, y \leq 511; \]

The difference between the absolute maximum and absolute minimum of the phase model is ~50 radians. In Fig. 1 pm(x, y) is shown.

The reference fringes \( I_0(x, y) \) are simulated as greyscale images with 8 bits per pixel with background intensity of 0.5 and amplitude modulation of 0.3. A certain and an avoidable degradation of any registered image is due to noise \( N(x, y) \). By ‘noise’ we mean any unwanted component of the image, including jitter, non-sinusoidal waveforms, speckle, etc. In the simulations we consider additive Gaussian or uniform noise; multiplicative noise; fringe deformation and stochastic jitter of CCD rows. Each noise can be considered separately or all of them cumulatively.

\[ I_0 = 255 \cdot \{0.5 + 0.3 \cdot \cos(w_o \cdot x)\} \]

\[ I_o = 255 \cdot \{0.5 + 0.3 \cdot \cos(w_o \cdot x + pm)\} + N(x, y) \]

where \( w_o = 2 \cdot \pi / T; \) T is the fringe period (in pixels). \( N(x, y) \) is the noise, additive in this case. To treat the case of fringes with constant global carriers, it is considered that fringes lie parallel to Oy axis. In Fig 2. the computer generated fringe pattern, corrupted with cumulative noises (with multiplicative noise) is shown.

The presented approach runs as follows:

1). Pre-process the input pattern. If multiplicative noise is present, make a homomorphic transformation [1]. Smooth the pattern with wavelet techniques. We do it with the 2 level discrete reverse bi-orthogonal wavelet [1, 2].

2). Phase evaluation is fulfilled with 1D Hilbert transformation on row-by-row basis. First, eliminate the background illumination by averaging, if the intensity is constant. We use also an adapted envelope approach for illumination with Gaussian distribution [2]. The four-quadrant arctangent function supplies the wrapped phase. We unwrap it, following Itoh approach [2, 3].

3). Smooth the estimated phase function by wavelet techniques. We use 4 level symlet wavelet [1] of 4-th order, followed by adapted Wiener filtering [1].

RESULTS AND DISCUSSIONS

The specific features of the noise models are as follows:

- Additive noise with Gaussian probability density function (PDF) with zero mean and standard deviation (STD) of +/- 10 intensity gray levels (out of 256)
- Multiplicative noise with Rayleigh PDF mean value of 1 and STD = 0.2732 (maximum for this PDF, which corresponds to two fully developed correlation cells within one pixel of the CCD detector)
- Non-sinusoidal modulation with second harmonic (ratio 2-nd to 1-st harmonic is equal to 0.25) and third harmonic (ratio 3-rd to 1-st harmonic is equal to 0.15)
- Noise due to stochastic high-frequency (environmental) vibrations with uniform PDF and values within +/- \( \pi/20 \) for each pixel, or
jitter noise for each row of the phase model (same PDF, same value)
The corrupted fringe pattern is processed with our approach. The presented results are for multiplicative noise. The obtained FM is 99.7%.
The difference between the model phase function and the evaluated phase is presented in Fig. 3.

![Fig. 3. Difference between model and evaluated phase function.](image)

In Fig. 4 we present the histogram of that difference. The result of de-noising and phase evaluation and unwrapping is very satisfactory.

![Fig. 4. Histogram of the difference model and evaluated phase function.](image)

Also, we extracted the phase with 1D complex Gabor wavelet and compared it to the Hilbert transform, keeping all the rest of the approach intact. The results are similar, but somewhat better for Hilbert (FM = 99.7% to 95.7% for Gabor).

We compared and analyzed the performance of different pre-processing algorithms and methods. To name a few: windowed Fourier filters, Frost filter, adaptive weight Wiener filter, anisotropic diffusion method [1, 2, 4]. Their performances were similar, but anisotropic diffusion had the highest figure of merit - FM = 99.8%.

**CONCLUSIONS**

We presented an approach for phase recovery from data, obtained from experiments in optical interferometry, by projection techniques, etc. The approach is based on a combination of Hilbert transform and different wavelet techniques for de-noising and smoothing. In our numeric simulations, we introduced different types of noise in order to get as close as possible to the physical reality. Our results on patterns with constant global angular carriers show that this approach is effective and robust. It is competitive with other methods, which are more complicated to implement and demand sophisticated software.

We intend to apply this approach for comparative analysis of single frame phase recovery and the well-known multiframe phase shifting algorithms.

**Acknowledgement:** Authors are grateful for the financial support provided by Bulgarian Science Fund under the project DCOST 01/7 and also acknowledge the COST Action MP1205.

**REFERENCES**

ВЪЗСТАНОВЯВАНЕ НА ФАЗАТА ОТ СТРУКТУРИ С ИВИЦИ: ПОДХОД, БАЗИРАН НА ХИЛБЕРТОВА ТРАНСФОРМАЦИЯ И ОБЕЗШУМЯВАНЕ ЧРЕЗ ВЪЛНИЧКИ

П. Шарланджиев, Н. Берберова, Д. Назърова и Г. Стоилов

Институт по оптически материали и технологии "Акад. Й. Малиновски", Българска Академия на науките, ул. "Акад. Г. Бончев", бл. 109, 1113 София, България

1Институт по механика, Българска академия на науките, ул. "Акад. Г. Бончев", блок 4, София ПС-1113, България

Постъпила на 10 октомври 2016 г.; коригирана на 10 ноември, 2016 г.

(Резюме)

Анализът на структури с ивици е важна задача в оптическата метрология. Ивични структури се формират чрез оптична интерферометрия, проекционни техники, припокриване на подобни вълнови полета и др. Структури с постоянна ъглова честота представляват прави ивици в полето на наблюдение. Изследваният обект деформира ивиците. Анализът на тези искривявания се нарича извличане на фазата и се използва широко в науката и техниката, напр. за получаване на топографията на 3Д обекти. Този анализ обикновено има 3 етапа: предварителна обработка (намаляване на шума в изображението, нулиране на фона), извличане на фазата (намиране на фазовото разпределение) и крайна обработка на резултатите (разопаковане на фазата, изглаждане, „почистване”).

В тази статия представяме подход, базиран на комбинация от преобразование на Хилберт за извличане на фазата и различни техники с вълници за обезшумяване и сглаждане на данните. Чрез числено моделиране показваме, че тази методология е ефективна и стабилна. Различни видове шум са моделирани и анализирани: гаусов добавъчен, интензитетно зависим мултипликативен (спекъл), джитер, наличие на високи хармонични в ивиците др.