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Study on the computing methods of mixed layer depth and gradation based on sand wave motion

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As for non-uniform sand mathematical models, the determination of mixed layer depth and gradation is of extremely important significance and will directly decide the reliability and accuracy of corresponding computing results. On the basis of sufficient indoor and outdoor data, the relation curve between relative wave height and relative velocity is represented, with curve parameters being matched; besides, based on the physical meanings of mixed layer depth and its relation with sand wave height, the sand wave motion-based computing method of mixed layer depth is derived out. Starting from the concept of mixed layer, it is assumed that the original bed material gradation will be uniform along depth, the porosity remains unchanged before and after scouring and silting, blending layer depth is a constant in the computing period, scoured sediment will be substituted equivalently by original bed materials below blending layer's lower boundary, deposited sediment will lead to equivalent ascending and descending of blending layer's upper and lower boundaries, the mixing will be uniform in the process, and the estimation methods of the sand wave bed's mixed layer under three models are derived out and obtained, including the pure deposition model, the pure scouring model and the coarse-deposition fine-scouring model.

Keywords: Sand wave,mixed layer, depth,gradation

INTRODUCTION

Mixed layer, also known as turbulent layer, exchange layer and mobile layer, refers to the bed layer under mobile or quasi-mobile state during the constant exchange with the mobile sediment in water flow within the scope below the bed surface during scouring. As for non-uniform sand mathematical models, the determination of mixed layer depth and gradation is of extremely important significance [1,2]. Firstly, from the aspect of starting conditions, the starting of non-uniform sand will not only be based on water flow conditions and the physical features of sediment particles, but will receive the effect of bed material's composition. Under different bed material composition, the difference of resistance features nearby the bed surface will lead to the difference of bed surface's velocity and shear force, while the difference of near-wall flow structure will result in the difference of concealing and exposure among coarse and fine particles, thus affecting the starting conditions of non-uniform sands. As a result, mixed layer gradation will greatly influence the starting situations of non-uniform sand, thus determining scoured strength of sediments with different particle sizes. Secondly, mixed layer depth directly determines the quantities of sediments participated in scour and exchange within the computing period, restrict the maximum scouring quantity in unit time and thus affect the scour velocity of the entire computing period. Thirdly, as mixed layer depth will determine the quantity of sediments participated in scouring and silting and exchange, with fixed scouring quantity or deposition quantity, mixed layer depth will directly affect the adjustment of bed material gradation of the entire mixed layer, further affecting sediment's starting and scouring strength. Finally, mixed layer depth will also determine the size of riverbed's extreme scouring quantity, as when the sediments within the entire mixed layer cannot be started or totally transformed into bed load, riverbed scouring will terminate or transform into the adjustment phase dominated by bed load motion. At the time, mixed layer depth will determine the quantity of sediments unable be started and under certain initial bed material gradation, the depth will also decide watercourse's final scouring quantity.

With respect to mixed layer depth, Chien N. [3] put forward in the coarsening study of Yellow River's downstream riverbed that the movable layer depth of a riverbed is about 2.5-3.5m. Zhao L. J. [4] also starting from the perspective of sand wave motion, divided the mixed layer into two parts, respectively as the direct exchange layer and the bed material adjustment layer. The probability method of the protection layer is on the basis of mixed layer's physical meanings, believes that the lower interface the mixed layer is the just bed surface elevation free of water flow influence and from the perspective of protection layer's generation, indirectly obtains

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mixed layer depth. The study of Broah [5] is representative as for this method. Li Y. T. [6] under given bed material composition and hydraulic conditions, analyzed the determination of the mixed mobile layer's lower interface under different sediment conditions and scouring time or scouring depth. Wang S. Q. [7] studied the adjustment of mixed mobile layer's bed material gradation during riverbed scouring and silting; research indicated that it was closely related with the mixing velocity, scale and other factors of mixed mobile layer's bed material; but the majority of existing computing methods cannot reflect the effects of mixed mobile layer's internal kinetic factors on the change of bed material gradation, thus greatly affecting the computing of unbalanced sediment of suspended load. In order to reflect the influence of mixed mobile layer's kinetic features on the change of bed material gradation, Liu J. M. [8] introduced the concept of "bed material's exchange velocity" related with the features of sand wave motion and set up corresponding computing models, and the introduction of this concept deepened the understanding about mixed mobile layer's mechanism. The mixed mobile layer equation set up by Armanini [9] considered the vertical mixing resulted from sand wave's cyclical fluctuations.

As an important riverbed form, sand wave will move in accompany with the exchange of bed materials and its motion can be regarded as a physical background for mixed layer computing. Considering from the physical background of sand wave motion, this paper has derived out the computing methods of mixed layer depth and gradation.

Sandwavemotion-based computing of mixed layer depth

Take sand wave motion as an example, during scouring, certain sediments in the mixed layer will be scoured away, while others will remain on the bed surface to maintain sand wave motion, which will continue to absorb the original bed material under static state on the original bed surface - all above constitute to the entire phenomena in the mixed layer.

With respect to sandy riverbed, the upper boundary of the mixed layer is the bed surface contacting with water flow, while the lower boundary is the static riverbed free of water flow turbulence. Actually, only the bed surface will be under the direct turbulence of water flow, which, however, will disturb the bottom of a wave trough by sand wave motion and in a wave cycle, the sediments on bed surface and those on the trough bottom will roll in turn within the range of one wave length. As a result, Karim and Kennedy proposed that mixed layer depth is the half of wave height, that is to say average wave height equals:

$$E_m = \frac{1}{2} h_s \tag{1}$$

Wherein: h_s is the wave height.

In the process of the flume experiment, Zhong D. Y. added dyed tracer sediments with the same specific gravity and the same gradation as that of bed materials into water flow and observed their mixing in bed material's exchange layer. Fig. 1 is the relationship between the maximum depth that tracer sediment in the riverbed and the sediment height. Judging from the figure, the mixing scale of bed material is related with sand wave height and can be expressed as

$$H_s = 0.7 h_s \tag{2}$$

That is to say, the sediment depth H_s is generally lower than h_s. The maximum value can be equal to h_s and mixed layer's average depth E_m is taken as the half of wave height, that is to say Equation (2) is suitable.

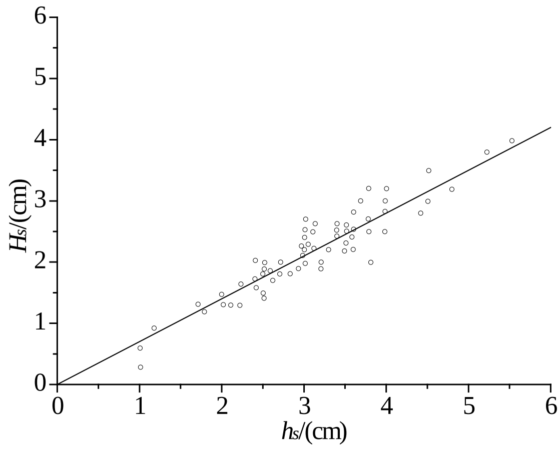

Fig. 1. Relationship BetweenSediment depth and Sand Wave Height.

Bed materials of sandy riverbed are basically movable and the coarsening layer is formed due to the repeated turbulence and repeated selective scouring of sediments in the mixed layer and is generally thicker than the mixed layer depth.

Fig. 2 and Fig. 3 are respectively the profiles and plans of sand wave height and coarsening layer depth on the bed surface observed in clean water scouring flume experiments. Judging from these figures:

$$E = \eta h_s > E_m \tag{3}$$

Wherein η is the proportionality coefficient, η=1.07-1.16 and the average value can be determined as 1.1.

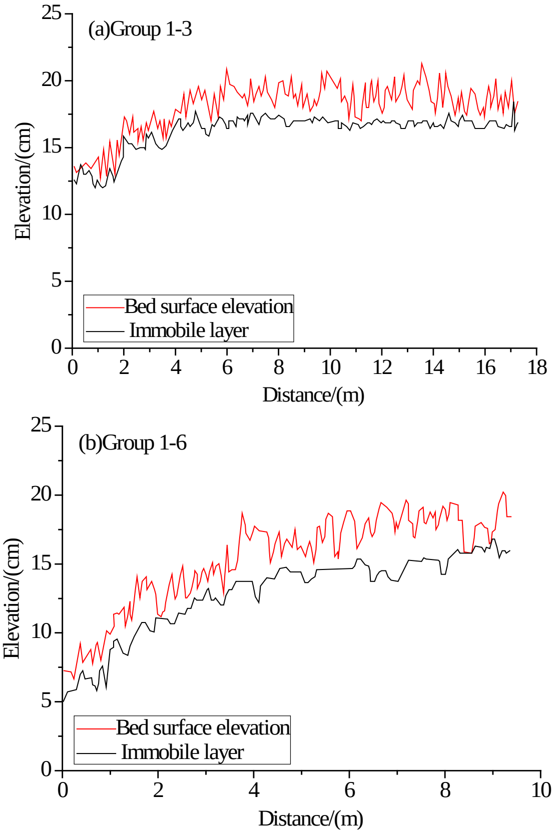

Fig. 2. Change of Coarsening Layer Depth at the Time of Experiment Ending.

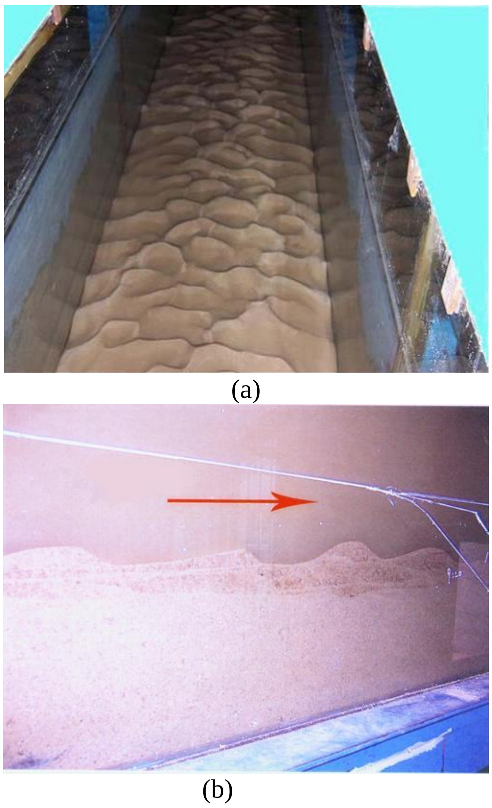

Fig. 3. Plan and Profile Pictures of a Sand Wave. (a) Plan, (b) Profile.

Yin X. L. [10] had put forward the idea that the coarsening layer depth formed due to fine-sand riverbed scouring was slightly larger than sand wave height as early as 1963 based on the measured materials concerning the downstream of Guanting Reservoir along the Yongding River and relevant flume experiments.

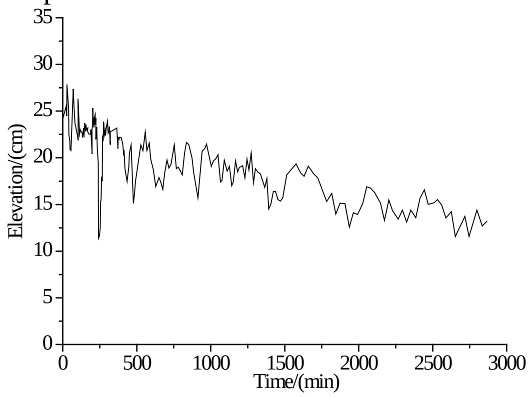

Fig. 4. Changing Process of Riverbed Elevation at the Same Position.

The reason that coarsening layer depth is larger than the sand wave height is that the water flow higher than sand peak will directly rush at sand wave's upstream face in the downstream along extremely unstable separation boundary and thus result in strong undermining on the upstream face; the sediments washed up will partially be rolled back to last sand wave's wave trough and partially led to the downstream by vertical water flow along the slope surface, leaving deep holes locally. With the downwards motion of sand waves on the upstream, these deep holes will be gradually filled up by the coarse particles rolled down from the peak top and the subsequent sand wave will then crawl thereon and generally will not disturb the hole bottom. Fig. 4 shows the change of certain fixed point's riverbed elevation under clean water scouring and the high point therein is the bed surface elevation at the time of wave peak, while the low point is that during wave trough; a 12cm deep hole will occur after 4 hours of scouring and then the sand wave will keep moving upwards and will not disturb the hole bottom; and until 45 hours of scouring when the accumulated undercut depth reaches 11cm, the trough bottom will be close to the hole bottom. This is why coarsening layer depth is larger than wave height, signifying that coarsening layer depth is generally larger than the mixed layer depth.

Sand wave height changes unstably with time, is extremely non-uniform in space and greatly different in plan, and cannot be easily measured, with limited study up to now. Earlier studies included the relationship between sand wave's relative height (h_s/h) and water flow's Froude number set up by Zhang R. J. [11] based on measured data concerning the Yangtze River. Afterwards, Reju [12] established the relationship between relative wave height (h_s/D) and bed load's sediment transport rate. Allen built up the relationship between sand wave's relative height (h_s/h) and non-dimensional shear stress. And Wang S. Q. [7] put forward the relationship between relative wave height (h_s/D_{50}) and effective scale-free shear force.

Karim and Kennedy suggested to compute the value of hs by the Allen equation, i.e.

$$h_s/h = b_0 + b_1\frac{\theta}{3} + b_2(\frac{\theta}{3})^2 + b_3(\frac{\theta}{3})^3 + b_4(\frac{\theta}{3})^4 \quad (4)$$

Based on measured values, b_0=0.079865, b_1=2.23897, b_2=-18.1264, b_3=70.9001 and b_4=-88.3283 and it is suitable to $0.25 \le \theta \le 1.5$.

The Zhang R. J. equation is as follows:

$$\frac{h_s}{h} = 0.086\frac{U}{\sqrt{gh}}(\frac{h}{d})^{\frac{1}{4}} \quad (5)$$

Substituting into the Somov equation, it will obtain:

$$\frac{h_s}{h} = 0.126\frac{U}{U_c}(\frac{d}{h})^{\frac{1}{12}} \quad (6)$$

Recently, Zhan Y. Z. [13] obtained the following wave height equation in combination with certain theoretical analysis based on the analysis results about the Yangtze River by Zhang B. N..

$$\frac{h_s}{h} = A - B(\frac{U}{U_c} - C)^2 \quad \frac{h_s}{h} = 0.126\frac{U}{U_c}(\frac{d}{h})^{\frac{1}{12}} \quad (7)$$

Wherein U_c is the starting velocity of sediment.A=0.53, B=0.1352and C=3.0.

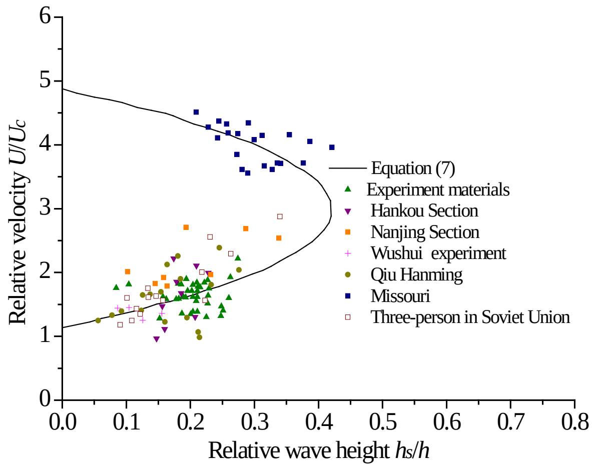

Fig. 5. Relationship BetweenRelative Wave Height and Relative Velocity.

Based on Equation (6) and (7), sand wave's relative height is mainly the function of water flow's relative strength U/U_c. Equation (7) signifies that: when water flow's relative strength is low, the relative wave height will increase with the increase of water flow's relative strength; while when water flow's relative strength is large, the relative wave height will decrease with the increase of water flow's relative strength, which accurately reflects the principles related with sand wave growth and decline and water flow strength. More extensive indoor and outdoor materials have been collected under this research, and the relation curve between relative wave height and relative velocity is drawn, as shown in Fig. 5. Results show that it would be more appropriate to describe relative wave height by Equation (7). Wherein A=0.42, B=0.12 and C=3.0 and compared with the coefficients obtained by Zhan Y. Z., A and$B$ are slightly different, while C is the same. The point-based trends in the figure are reasonable. The relatively scattered phenomenon is probably resulted from measurement accuracy or the non-corresponding between sand wave scale and water flow conditions, which will lead to certain lagging behind.

Estimation of sand wave-free bed's blending layer gradation

Yue P. J. [14,15] assumed that the distribution of original bed material gradation along depth is constant and that the porosity remains unchanged before and after scouring and silting, and also divided bed material gradation into three categories based on riverbed's scouring and silting features, i.e.

One-way deposition

$$p_i = \Delta h_i'/\Delta h' \quad (8)$$

One-way scouring

$$p_i = \frac{p_{0i}(T+\Delta T) - p_i''\Delta h''}{T+\Delta T-\Delta h''} \quad (9)$$

Coarse-deposition fine-scouring

$$p_i = \frac{p_{0i}(T+\Delta T) + p_i'\Delta h' - p_i''\Delta h''}{T+\Delta T+\Delta h'-\Delta h''} \quad (10)$$

Wherein T is the initial blending layer depth at the first time period; p_{0i} is original bed material gradation, while p' and p'' are respectively the gradation of the deposit and the scoured material; ΔT is the depth of the turbulent layer to be substituted from former period's riverbed in the next time period and to ensure that no negative value will occur concerning the turbulent layer gradation in the period of scouring, it needs to satisfy:

$$p_{0i}T - p_i''\Delta h'' \ge 0 \quad (11)$$

Equation (8) - (10) are only appropriate to sand wave-free beds. For sand wave bed, Equation (8) and (9) are both inaccurate and while the other equations can satisfy the sediment balance requirements, the denominator therein should be E_m.

Estimation of sand wave bed's blending layer gradation

Regarding the concept of blending layer, it is assumed that original bed material gradation p0i is uniformly distributed along depth; the porosity remains unchanged before and after scouring and silting; the blending layer depth is a constant in the computing period; scoured sediment will be substituted equivalently by original bed materials below blending layer's lower boundary; deposited sediment will lead to equivalent ascending and descending of blending layer's upper and lower boundaries; the mixing will be uniform in the process, etc. Under above conditions,

(1) Pure deposition

Pure deposition is as shown in Fig. 6 and the equation of sediment balance in the blending layer is:

$$p_i(n-1)(E_m-\Delta h'(n))+p_i'(n)\Delta h'(n)=p_i(n)E_m \quad (12)$$

Namely:

$$p_i(n)=p_i(n-1)-[p_i(n-1)-p_i'(n)]\frac{\Delta h'(n)}{E_m} \quad (13)$$

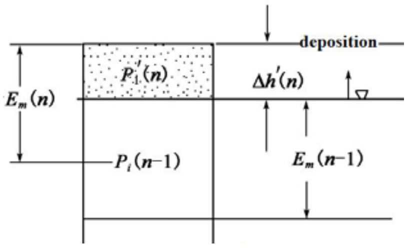

Fig. 6. ShematicClumnarPofile of Pre Dposition.

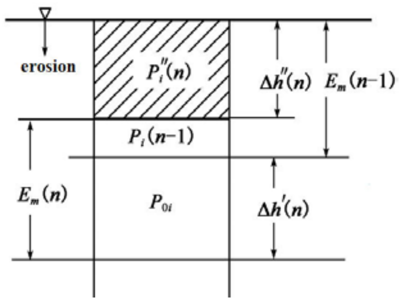

Fig. 7. Schematic Columnar Profile of Pure Scouring.

The time step taken in computing is not proper to be over large and it needs to make sure $\Delta h'(n)/E_m<1$, otherwise, the bed material gradation will be distorted.

(2) Pure scouring

Pure scouring is as shown in Fig. 7 and the equation of sediment balance in the blending layer is:

$$E_m p_i(n-1)-p_i''(n)\Delta h''(n)+p_{0i}\Delta h''(n)=E_m p_i(n) \quad (14)$$

Based on Equation (14), it could obtain that

$$p_i(n)=p_i(n-1)+[p_{0i}-p_i''(n)]\frac{\Delta h''(n)}{E_m} \quad (15)$$

(3) Coarse-deposition fine-scouring

Coarse-deposition fine-scouring is generally found on the downstream away from the dam, the sediments recovered by upstream riverbed are relatively coarser and those bed materials downstream finer; in the process of sediment exchange, the coarse-deposition fine-scouring model occurs. Under the limitation of sediment carrying ability and the suspension probability, the deposition quantity of coarse sands $\Delta h'$ is always less than the fine sands exchanged out $\Delta h''$, while the riverbed will still undercut ($\Delta h''$-$\Delta h'$). As shown in Fig. 8, the equation of sediment balance in the blending layer is:

$$E_m p_i(n-1)-p_i''(n)\Delta h''(n)+p_i'(n)\Delta h'(n)+p_{0i}[\Delta h''(n)-\Delta h'(n)]=E_m p_i(n) \quad (16)$$

Namely

$$p_i(n)=p_i(n-1)+[p_{0i}-p_i''(n)]\frac{\Delta h''(n)}{E_m}-[p_{0i}-p_i'(n)]\frac{\Delta h'(n)}{E_m} \quad (17)$$

In Equation (17), when $\Delta h'$=0, it will obtain Equation (15); while when $\Delta h''$=0, it will obtain the Equation (13) and as it is deposit, the p_{0i} here shall be substituted by $p_{i(n-1)}$.

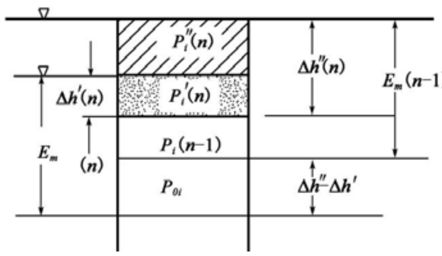

Fig. 8. Schematic Columnar Profile of Coarse-deposition Fine-scouring.

CONCLUSION

As for non-uniform sand mathematical models, the determination of mixed layer depth and gradation is of extremely important significance and will directly decide the reliability and accuracy of corresponding computing results. On the basis of sand wave motion, the computing of mixed layer depth and gradation are studied in this paper.

(1) On the basis of sufficient indoor and outdoor data, in combination with relevant flume experiment data, the relation curve between relative wave height and relative velocity is represented in this paper, with curve parameters being matched; besides, the principles related with sand wave growth & decline and water flow strength are exposed.

(2) Regarding the motion of bed surface's sand wave as a physical background for mixed layer

computing and on the basis mixed layer depth's physical meanings and its relation with sand wave height, the sand wave motion-based computing method of mixed layer depth is derived out.

(3) Starting from the concept of mixed layer, it is assumed that the original bed material gradation will be uniform along depth, the porosity remains unchanged before and after scouring and silting, blending layer depth is a constant in the computing period, scoured sediment will be substituted equivalently by original bed materials below blending layer's lower boundary, deposited sediment will lead to equivalent ascending and descending of blending layer's upper and lower boundaries, the mixing will be uniform in the process, etc., and the estimation methods of the sand wave bed's mixed layer under three models are derived out and obtained, including the pure deposition model, the pure scouring model and the coarse-deposition fine-scouring model.

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