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Effect of porosity on the flow with heat transfer of a non-Newtonian power law fluid due to a rotating disk with uniform suction and injection

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The steady flow through a porous medium of an incompressible non-Newtonian power law fluid due to the uniform rotation of a rotating disk of infinite extent is studied with heat transfer. The disk is immersed in a porous medium that is assumed to obey Darcy's model while a uniform injection or suction is applied through its porous surface. A numerical solution for the governing nonlinear differential equations is obtained. The effect of characteristics of the non-Newtonian fluid, the porosity of the medium and the suction or injection velocity on the velocity and temperature distributions is analyzed.

Keywords: Rotating disk flow, non-Newtonian fluid, power law fluid, porous medium, heat transfer, numerical solution.

NOMENCLATURE

D_{ij} : rate of strain tensor,
Ec: Eckert number,
(*F*, *G*, *H*): non-dimensional velocity components,
k: thermal conductivity,
K: consistency coefficient,
*K*1: Darcy permeability,
m: porosity parameter,
n: power-law index,
Nu: Nusselt number,
p: pressure gradient,
P: non-dimensional pressure,
p_∞ : pressure of the ambient fluid,
Pr: Prandtl number,
Q: rate of heat transfer,
T: temperature of the fluid,
T_w: temperature of the disk,
T_∞ : temperature of the ambient fluid,
S: suction parameter,
w_o: vertical velocity at the disk,
$\vec{V}$: velocity vector,
(*u*, *v*, *w*): velocity components,
(r, φ , z): cylindrical coordinates,
θ : non-dimensional temperature,
μ: viscosity of the fluid,
ν : kinematic viscosity of the fluid,
ρ: density of the fluid,
ω : angular velocity of the disk,
τ : stress tensor,
ζ : non-dimensional distance.

INTRODUCTION

The mathematical formulation of the steady fluid flow due to the rotation of a disk with infinite extension was obtained by von Karman in 1921 [1]. A similarity transformation was introduced which reduced the governing partial differential equations to ordinary differential equations. Later, Cochran [2] obtained an asymptotic solution for the resulting system of ordinary differential equations. The extension of the steady state problem to the transient state was carried out by Benton [3]. The heat transfer from a rotating disk maintained at a constant temperature was studied by many authors under different assumptions [4,5]. Attia [6] extended the problem discussed in [4,5] to the transient state considering an electrically conducting fluid and in the presence of an applied uniform magnetic field. The practical importance of non-Newtonian fluids attracted the attention of many authors to study their rule in rotating disk flow. The steady flow of a non-Newtonian fluid due to a rotating disk with uniform suction was considered by Mithal [7]. Mitschka [8] and Mitschka and Ulbrecht [9] extended the von Karman analysis to non-Newtonian of the power-law type fluids. Motion of power-law fluids in the presence of a magnetic field has been studied earlier by several authors [10-12]. Examples of non-Newtonian fluids which might be conductors of electricity were given by Sarpkaya [13], flow of nuclear slurries and of mercury amalgams, and lubrication with heavy oils and greases. The problem studied by Mitschka [8] and Mitschka and Ulbrecht [9] was reconsidered in [14] with a

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particular view to address the reliability of their numerical solutions of the extremely non-linear ordinary differential equations arising in the presence of a non-linear rheological equation of state. The magnetohydrodynamic flow of a power-law fluid over a rotating disk is of particular interest since the magnetic force field in this case vanishes outside the viscous boundary layer and therefore affects the fluid motion only within the boundary layer. The effect of an externally applied magnetic field on the flow of a power-law fluid due to a rotating disk, and in particular how effectively a uniform magnetic field can be utilized as a means of flow control, was studied [15-17]. Batista [18] obtained an analytical solution of the Navier–Stokes equations for the case of the steady flow of an incompressible fluid between two uniformly co-rotating disks. Turkyilmazoglu [19] obtained an exact solution for the flow of a viscous hydromagnetic fluid due to the rotation of an infinite disk in the presence of an axial uniform steady magnetic field with the inclusion of Hall current effect. Nazir and Mahmood [20] studied the effects of disks contracting, rotation and heat transfer on the viscous fluid between heated contracting rotating disks. Bachok et al. [21] studied the steady flow of an incompressible viscous fluid due to a rotating disk in a nano fluid. Devi and Devi [22] studied the thermal radiation effect over an electrically conducting, Newtonian fluid in a steady laminar magnetohydrodynamic convective flow over a porous rotating infinite disk with consideration of heat and mass transfer. Ming et al. [23] studied the steady flow and heat transfer of a viscous incompressible power-law fluid over a rotating infinite disk. Assuming that the thermal conductivity follows the same function as the viscosity, the governing equations in the boundary layer are transformed into a set of ordinary differential equations by a generalized Karman similarity transformation. The corresponding nonlinear two-point boundary value problem was solved by the multi-shooting method. Attia [24], Sahoo [25] and Osalusi [26] provided some researches about the Reiner–Rivlin model. Urkyilmazoglu [27] obtained exact solutions to the steady Navier–Stokes equations for the incompressible Newtonian viscous electrically conducting fluid flow motion due to a disk rotating with a constant angular speed. The rotating disk flow through a porous medium that obeys Darcy's model was carried out by Attia [28,29].

In the present paper, the steady laminar flow through a porous medium of an incompressible viscous non-Newtonian power-law fluid due to the uniform rotation of a disk of infinite extent in the presence of uniform suction and injection is studied with heat transfer. A uniform injection or suction is applied perpendicularly through the surface of the disk which is maintained at a constant temperature. The flow in the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [30,31]. The governing nonlinear differential equations for the flow and temperature distributions are solved numerically using the method of finite differences. The effect of the porosity of the medium, the characteristics of the non-Newtonian power-law fluid and the suction or injection velocity on the steady flow and temperature fields is presented and discussed.

BASIC EQUATIONS

Let the disk lies in the plane z=0 and the space z>0 is occupied by an incompressible viscous non-Newtonian power law fluid as shown in Fig.1.

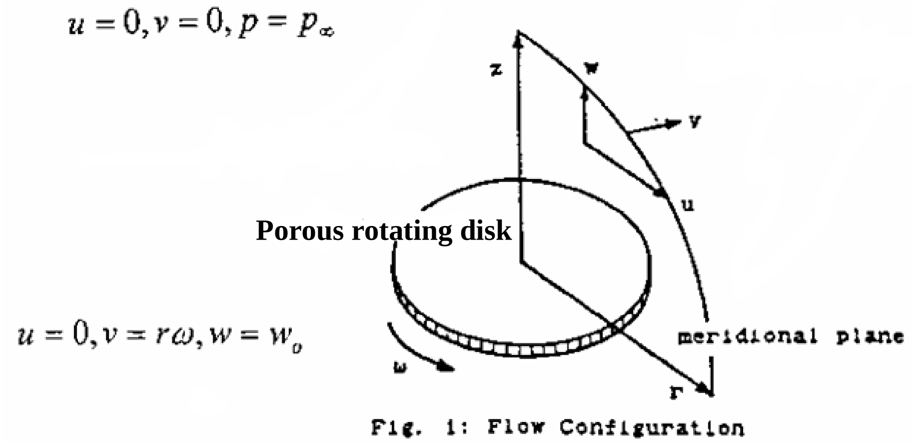


Fig.1. Flow configuration

The motion is due to rotation of an insulated disk of an infinite extent about an axis perpendicular to its plane with constant angular speed ω. Otherwise the fluid is at rest under pressure p_∞. The disk is maintained at a constant temperature T_w. A uniform injection or suction is applied at the surface of the disk for the entire range from large injection velocities to large suction velocities. The steady flow induced by a rotating disk immersed in a porous medium where the Darcy's model is assumed [30,31]. u, v and w the velocity components in the radial, azimuthal and vertical directions, respectively.

The non-Newtonian fluid considered in the present work is the power law model, where, the stress τ is related to the rate of strain tensor D_{ij} as [7].

$$\tau = 2\mu D = 2K(2D_{ij}\, D_{ji})^{(n-1)/2} D \qquad (1)$$

where, D denotes the deformation or rate of strain, tensor and K and n are the consistency coefficient and the power law index, respectively.

The equations of steady motion are given by the continuity equation

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad (2)$$

and the momentum transfer equations

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) = \frac{\partial \tau_r^r}{\partial r} + \frac{\partial \tau_r^z}{\partial z} + \frac{\tau_r^r - \tau_\varphi^\varphi}{r} - \frac{\mu}{K1}u \qquad (3)$$

$$\rho\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right) = \frac{\partial \tau_\varphi^r}{\partial r} + \frac{\partial \tau_\varphi^z}{\partial z} + \frac{2\tau_\varphi^r}{r} - \frac{\mu}{K1}v \qquad (4)$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = \frac{\partial \tau_z^r}{\partial r} + \frac{\partial \tau_z^z}{\partial z} + \frac{\tau_z^r}{r} - \frac{\mu}{K1}w \qquad (5)$$

where, ρ is the density of the fluid. The boundary conditions are given as

$$u = 0\,,\; v = r\omega,\; w = w_o \text{ at } z = 0 \qquad (6a)$$

$$u \to 0,\; v \to 0,\; p \to p_\infty \text{ as } z \to \infty \qquad (6b)$$

where $K1$ is the Darcy permeability [30-31]. Eq. (6a) indicates the no-slip conditions of viscous flow applied at the surface of the disk and ensures that the convective velocity normal to the surface of the disk specifies the mass injection or withdrawal. Due to the uniform suction or injection, the vertical velocity component takes a constant non-zero value w_o at z=0. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in Eq. (6b).

Due to the difference in temperature between the wall and the ambient fluid, heat transfer takes place. The energy equation takes the form [5];

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\frac{\partial^2 T}{\partial z^2} + \mu\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right) + \frac{k}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial T}{\partial r}\right) \qquad (7)$$

where, T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that, by continuity considerations, the temperature equals T_w at the surface of the disk. At large distances from the disk, T tends to T_∞ where T_∞ is the temperature of the ambient fluid.

We introduce von Karman transformations adapted for power-law fluid flow [1,15],

$$u = r\omega F(\zeta),\; v = r\omega G(\zeta),$$

$$w = \left(\frac{\omega^{1-2n}}{K/\rho}\right)^{-1/(1+n)} r^{(n-1)/(n+1)} H(\zeta),$$

$$\zeta = z\left(\frac{\omega^{2-n}}{K/\rho}\right)^{1/(1+n)} r^{(1-n)/(n+1)},$$

$$\theta = (T - T_\infty)/(T_w - T_\infty)$$

where, ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and θ are non-dimensional functions of ζ, and ν is the kinematic viscosity of the fluid, $\nu = \mu/\rho$. With these definitions, Eqs. (2)-(7) take the form

$$H' + 2F - \left(\frac{1-n}{1+n}\right)\zeta F' = 0 \qquad (8)$$

$$F^2 + G^2 + \left(H + \left(\frac{1-n}{1+n}\right)\zeta F\right)F' + mF = \left(((F')^2 + (G')^2)^{(n-1)/2} F'\right)' \qquad (9)$$

$$2FG + \left(H + \left(\frac{1-n}{1+n}\right)\zeta F\right)G' + mG = \left(((F')^2 + (G')^2)^{(n-1)/2} G'\right)' \qquad (10)$$

$$F = 0,\; G = 1,\; H = S \text{ at } \zeta = 0 \qquad (11a)$$

$$F \to 0,\, G \to 0 \text{ as } \zeta \to \infty \qquad (11b)$$

$$\frac{1}{\Pr}\theta'' - H\theta' + Ec\left((F')^2 + (G')^2\right)^{(n+1)/2} = 0 \qquad (12)$$

where, prime denotes differentiation with respect to ζ, $\Pr = \mu_0^{2/(n+1)} c_p (\omega^3 r^2)^{\frac{n-1}{n+1}}/k$ is the Prandtl number, $Ec = \omega^2 r^2 / c_p (T_w - T_\infty)$ is the Eckert number and $m = \nu / K\omega$ is the porosity parameter. The boundary conditions for the velocity problem are given by eqs 11-12 where, $S = w_o/\sqrt{\omega\nu}$ is the uniform suction or injection parameter which takes constant negative values for suction and constant positive values for injection. The boundary conditions in terms of θ are expressed as

$$\theta(0) = 1\,,\; \theta \to 0 \text{ as } \zeta \to \infty \qquad (13)$$

The above system of Eqs. (8)-(13) is sufficient to solve for the three components of the flow velocity.

The heat transfer from the disk surface to the fluid is computed by application of Fourier's law

$$Q = -k\left(\frac{dT}{dz}\right)_w$$

Introducing the transformed variables, the expression for Q becomes

$$Q = -k(T_w - T_\infty)\sqrt{\frac{\omega}{\nu}}\theta'(0)$$

By rephrasing the heat transfer results in terms of a Nusselt number defined as, $Nu = Q\sqrt{\omega/\nu}\,/\,k(T_w - T_\infty)$ the latter equation becomes

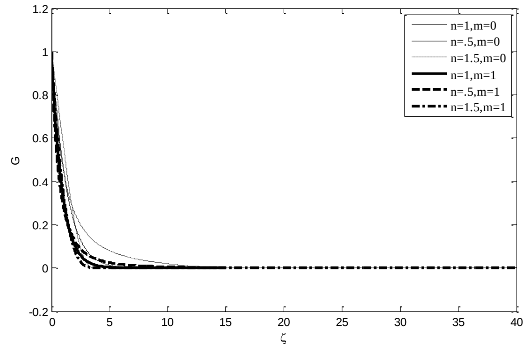
$$Nu = -\theta'(0) \qquad (14)$$

The system of non-linear ordinary differential equations (8)-(13) is solved for the three components of the flow velocity and temperature, using the Crank-Nicolson implicit method [32]. The resulting system of difference equations has to be solved in the infinite domain $0<\zeta<\infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for $30\le\zeta_\infty\le40$ for $n \le 1$, $8\le\zeta_\infty\le12$ for $n > 1$ and step size $\Delta\zeta=0.001$ which are found adequate for the ranges of the parameters studied here.

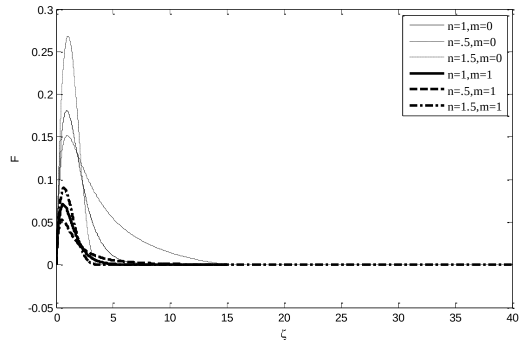
RESULTS AND DISCUSSION

Figs 2(a,b,c,d) present the steady state velocity components and temperature, G, F, H, and θ, respectively, for various values of the power-law index n=0.5, 1, 1.5 and the porosity parameter m=0, 1.

In these figures, S=0, Ec=0.2 and Pr=0.72. For the Newtonian case, the azimuthal velocity component G decays rapidly with the vertical distance from the disk as shown in Fig. 2a. Consequently, the centrifugal force produces an outward flow in the radial direction with velocity F which is compensated by an axial inflow towards the rotating disk with velocity H as depicted in Figs. 2b and 2c. Also Fig. 1a shows that the rate of decaying decreases for the case of shear-thinning fluids (n<1) whereas it increases for the case of shear-thickening fluids (n>1).



(a)



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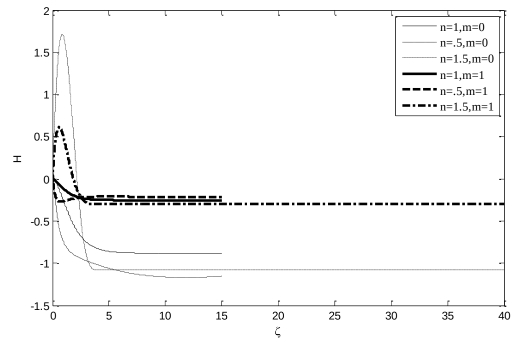

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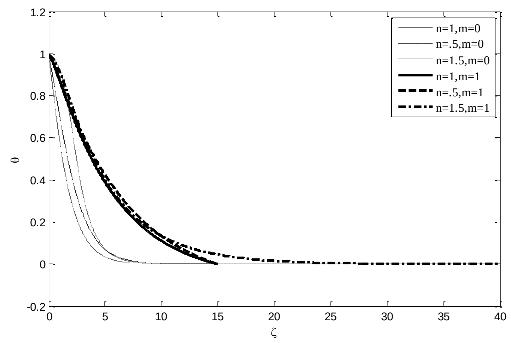


(d)

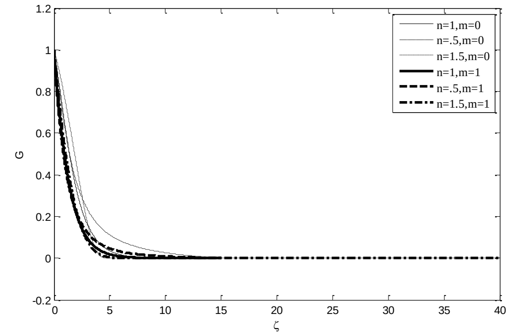
Fig. 2 Variation of G, F, H, and θ for various of n and m and for S=0, Ec=0.2 and Pr=0.72

Figure 2b indicates an interesting effect on the velocity component F for the case of shear-thinning fluids (n<1) is that, although the location of the peak in the radial velocity component F remains the same as in the Newtonian case, the shear-driven fluid motion in the plane parallel to the disk tends to penetrate further into the otherwise stagnant fluid than for the case of Newtonian fluid. Consequently, this leads to the thickening of the boundary layer which gives rise to a significantly enhanced inflow in the axial direction. Another interesting result presented in Fig. 2c which is the appearance of peaks in the axial velocity components for the non-Newtonian case near the surface of the disk with the presence of cross-over points in the profiles of *H* for the case of shear-thickening fluids (n>1) which becomes more apparent for the non-porous case. We conclude that the effect of non-Newtonian fluid characteristics on the axial flow towards the disk depends on the porosity parameter. Figs. 2(a,b,c) show that only for the case of shear-thinning fluids (n<1), the velocity components F and G in the plane parallel to the disk reach their asymptotic zero values closer to the disk than the axial velocity component H reaches its asymptotic infinite value. It is clear from Figs. 2(a,b,c) that the porosity of the medium has a damping effect on the three velocity components G, F, and H which is expected.

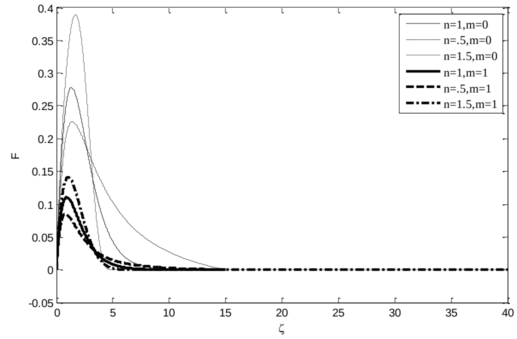
Figure 2d indicates that increasing the porosity parameter m increases θ due to the effect of the porosity in damping the axial flow towards the disk and, consequently, prevents bringing the fluid at a near-ambient temperature towards the surface of the disk in addition to the effect of Joule dissipation.

It is indicated in all Figs. 2(a,b,c,d) that the effect of the porosity of the medium on the velocity components and temperature is more pronounced for the case of shear-thinning fluids (n<1) than the case of shear-thickening fluids (n>1). Figure 2d depicts the effect of increasing the power-law index n in increasing θ in the non-porous case due to its effect in decreasing the axial flow towards the disk. On the other hand, in the porous case, increasing the power-law index n decreases θ due to its effect in increasing the axial flow towards the disk which helps bringing the fluid at a near-ambient temperature towards the surface of the disk. We conclude that, the effect of non-Newtonian fluid characteristics on the temperature distribution depends on the porosity parameter.

Figs. 3(a,b,c,d) and 4(a,b,c,d) present the influence of the axial flow at the surface of the disk for injection (S=1) and suction (S=-1) respectively, on the steady state velocity components and temperature, F, G, H, and θ, respectively, for various values of the power-law index n=0.5, 1, 1.5 and the porosity parameter m=0, 1.



(a)



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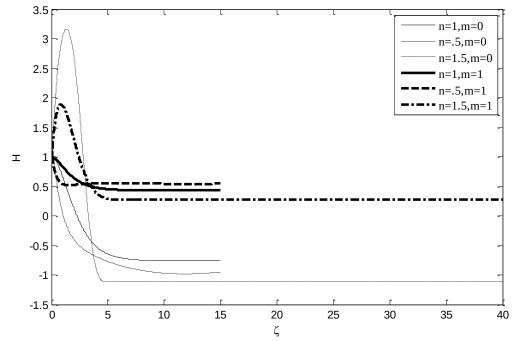

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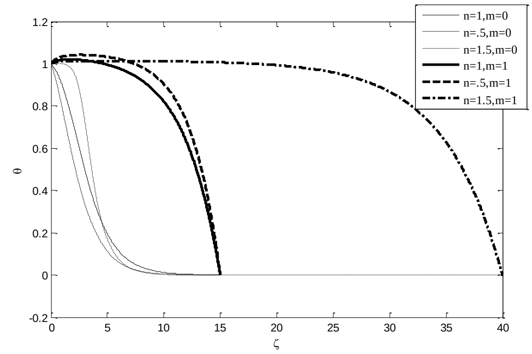
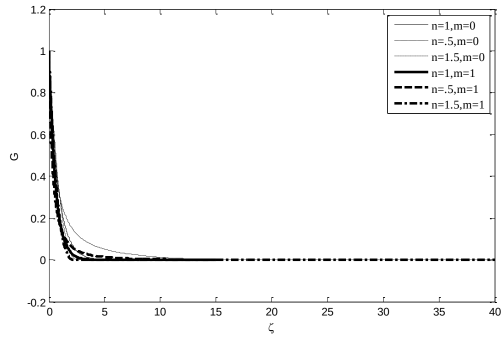
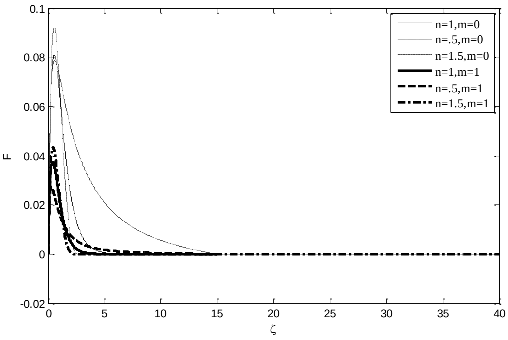


(d)

Fig. 3. Variation of F, G, H and θ for various n and m and for S=1, Ec=0.2, and Pr=0.72.



(a)



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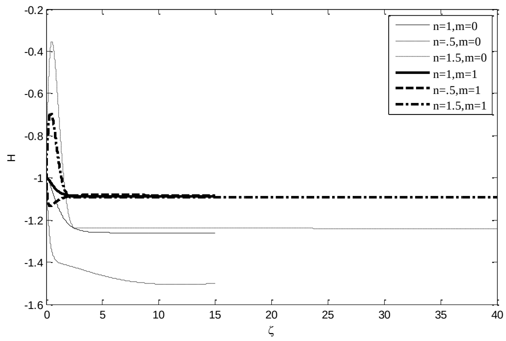

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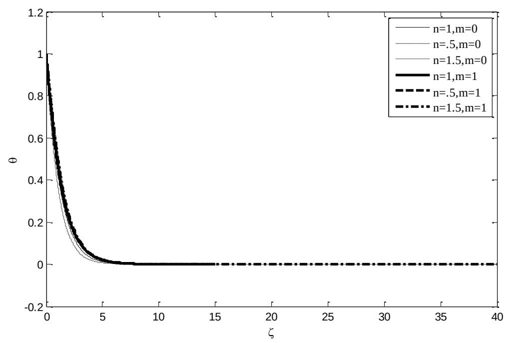

(d)

Fig. 4 Variation of *G,F,H* and θ for various of n and m and for *S*=-1, *Ec*=0.2 and *Pr*=0.72.

In these figures, *Ec*=0.2, *Pr*=0.72. It is clear from Figs. 3b and 4b that increasing the suction velocity leads to a rapid decrease in *F* while increasing the injection velocity increases *F*. The effect of the porosity of the medium and the non-Newtonian fluid characteristics on the radial and azimuthal flows is more pronounced in the case of injection than that in the case of suction as shown in Figs. 3(b,c) and 4(b,c).

Figure 3c indicates that increasing the injection velocity reduces the axial flow towards the disk while with increasing injection velocity, the outflow penetrates to greater distances from the disk surface. Figure 4c shows that increasing the suction velocity increases the axial flow towards the disk while the magnitude of the axial velocity at infinity is larger than that at the disk. Figs. 3d and 4d indicate the effect of the fluid injection in decreasing the temperature significantly by blanketing the surface with fluid whose temperature is close to the wall temperature. Suction has an opposite effect on the temperature, since fluid at near-ambient temperature is brought to the neighborhood of the surface of the disk. The influence of the porosity and the flow index on the temperature distribution is more apparent in the case of injection than in the case of suction.

Table 1 presents the variation of the radial wall shear *F'(0)*, azimuthal wall shear *G'(0)*, the axial inflow at infinity H(∞) and the Nusselt number at the surface of the disk θ'(0) for various values of the parameter *n*, *m* and respectively, for *S*=0, *Ec*=0.2 , *Pr*=0.72.

It is clear that increasing *n* increases the radial wall shear while decreases the azimuthal wall shear and its effect is more apparent for the non-porous case. On the other hand, the influence of the parameter *n* on the axial inflow at infinity depends on the porosity parameter. Increasing *n* decreases the axial inflow towards the disk in the non-porous case, while slightly increases it in the porous case. It is clear from the results presented in Table 1 that increasing the parameter *n* decreases the heat transfer at the surface of the disk and consequently decreases the Nusselt number *Nu*. Increasing the porosity parameter *m* decreases the radial wall shear, the axial inflow at infinity, the heat transfer at the surface of the disk, but increases the azimuthal wall shear. Tables 2 and 3 present the variation of the radial wall shear, azimuthal wall shear, the axial inflow at infinity, the Nusselt number at the surface of the disk for various values of the parameter *n*, *m* and, respectively, for *S*=1 and *S*=-1 and for *Ec*=0.2 , *Pr*=0.72.

It is interesting to see the reversal of the sign of the axial inflow at infinity and the heat transfer coefficient in the injection case. Also, it is depicted that the influence of the porosity on both the radial and the azimuthal wall shears becomes more apparent for the case of injection (*S*=1) then the case of suction (*S*=-1).

Table 1 Variation of $F'(0)$, $G'(0)$, $H(\infty)$, $\theta'(0)$ for various values of n and m and for S=0, Pr=0.72 and Ec=0.20.

Power Law Index(n)	$F'(0)$		$-G'(0)$		$-H(\infty)$		$-\theta'(0)$	
	m=1	m=0	m=1	m=0	m=1	m=0	m=1	m=0
1.5	0.5458	0.3166	0.563	0.9606	0.7349	0.2556	0.2135	0.0404
1.3	0.5313	0.3131	0.5804	0.992	0.7704	0.2553	0.2339	0.0407
1.1	0.517	0.3103	0.6024	1.0378	0.8334	0.2543	0.261	0.0409
1	0.5102	0.3093	0.6159	1.0691	0.8845	0.2533	0.2761	0.0089
0.9	0.4938	0.3062	0.6696	1.127	0.9134	0.244	0.368	0.0187
0.8	0.4794	0.3041	0.7294	1.2005	0.989	0.2344	0.4732	0.030
0.6	0.4583	0.3041	0.879	1.4300	1.2694	0.2125	0.6842	0.0526
0.5	0.4523	0.3074	0.9813	1.6228	1.4087	0.1977	0.7719	0.058

Table 2 Variation of $F'(0)$, $G'(0)$, $H(\infty)$, $\theta'(0)$ for various values of n and m and for S=1, Pr=0.72 and Ec=0.20.

Power Law Index(n)	$F'(0)$		$-G'(0)$		$-H(\infty)$		$-\theta'(0)$	
	m=0	m=1	m=0	m=1	m=0	m=1	m=0	m=1
1.5	0.5393	0.3466	0.2605	0.6541	0.5968	-0.3897	0.0327	-0.0886
1.3	0.5214	0.3368	0.277	0.6652	0.6296	-0.4048	0.0397	-0.0928
1.1	0.5008	0.3267	0.2939	0.6806	0.6694	-0.422	0.0497	-0.0976
1	0.4895	0.3217	0.3022	0.6907	0.7607	-0.4317	0.0546	-0.1077
0.9	0.4748	0.3152	0.337	0.7175	0.7824	-0.4621	0.1135	-0.1146
0.8	0.4574	0.3088	0.3714	0.749	0.8637	-0.4923	01973	-0.1224
0.6	0.4156	0.2963	0.4333	0.8327	1.1377	-0.5545	0.3905	-0.1409
0.5	0.3903	0.2902	0.4575	0.8904	1.2558	-0.590	0.4711	-0.1513

Table 3 Variation of F'(0), G'(0), H (∞), θ'(0) for various values of n and m and for S=-1, Pr=0.72 and Ec=0.20.

Power Law Index(n)	$F'(0)$		$-G'(0)$		$-H(\infty)$		$-\theta'(0)$	
	m=0	m=1	m=0	m=1	m=0	m=1	m=0	m=1
1.5	0.3613	0.2345	1.0519	1.3672	1.1903	1.0868	0.6655	0.5779
1.3	0.3691	0.2396	1.0854	1.4492	1.2072	1.0878	0.681	0.5835
1.1	0.3811	0.2465	1.1374	1.5712	1.2365	1.0891	0.7022	0.5900
1	0.3896	0.251	1.1752	1.6571	1.2606	1.0898	0.7156	0.5933
0.9	0.3941	0.255	1.2544	1.7894	1.2864	1.0889	0.772	0.6165
0.8	0.4026	0.2604	1.3622	1.9584	1.3346	1.0878	0.8375	0.6408
0.6	0.4395	0.2795	1.7426	2.5783	1.5402	1.0842	0.9842	0.6869
0.5	0.479	0.2971	2.1089	3.1979	1.6635	1.0802	1.0522	0.7026

CONCLUSIONS

In this paper the steady flow through a porous medium of a non-Newtonian fluid induced by a rotating disk with heat transfer was studied in the presence of a uniform suction and injection. The effect of the porosity parameter, the non-Newtonian fluid characteristics and the uniform suction or injection velocity on the velocity and temperature distributions was considered.

It is interesting to find the appearance of peaks in the axial velocity components for the non-Newtonian case near the surface of the disk with the presence of cross-over points in the profiles of H in the porous case. It is depicted that the effect of the non-Newtonian fluid characteristics on the axial flow towards the disk and temperature distribution depends on the porosity parameter. It is depicted that the influence of the porosity on both the radial and azimuthal wall shears becomes more apparent for the cases of suction or injection. It is concluded that the influence of the porosity and the flow index on the temperature distribution is more apparent in the case of injection than the case of suction. It is interesting to see the reversal of the sign of the axial inflow at infinity and the heat transfer coefficient in the injection case.

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ЕФЕКТ НА ПОРЬОЗНОСТТА ВЪРХУ ПОТОК С ПРЕНОС НА ТОПЛИНА НА НЕНЮТОНОВ ФЛУИД, ДЪЛЖАЩ СЕ НА ВЪРТЯЩ СЕ ДИСК С ПОСТОЯННО ВСМУКВАНЕ И ИНЖЕКТИРАНЕ

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(Резюме)

Беше изследван устойчивият поток, дължащ се постоянното въртене на въртящ се диск с безкрайна големина с пренос на топлина, на несвиваем ненютонов флуид през пореста среда. Дискът е потопен в порьозна среда, която се предполага, че се подчинява на модела на Дарси докато постоянното инжектиране или всмукване се прилага чрез порестата му повърхност. Получено е числено решение за ръководните нелинейни диференциални уравнения. Ефектът на характеристиките на не-Нютоновия флуид, порьозността на средата и скоростта на засмукване или инжектиране върху разпределенията на скоростта и температурата са анализирани.