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# Experimental determination of coefficient of soil hydrodynamic dispersion

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The unsaturated soil hydrodynamic dispersion test was done by using unsaturated silt loam in the Yangtze River Estuary. The hydrodynamic dispersion coefficient was expressed as a linear function form in the form of mechanical dispersion coefficient. Based on the mass conservation principle, we deduced the new formulas to calculate coefficient of hydrodynamic dispersion, and calculated the hydrodynamic dispersion coefficient of unsaturated silt loam according to the soil water and salt dynamic data measured by the vertical soil column method. Based on the calculated results, the relationship between hydrodynamic dispersion coefficient and the pore velocity were established. The experimental results show that there is a linear relationship between the hydrodynamic dispersion coefficient and the average pore water velocity, and the calculated results are close to the previous ones. The method is proved to be correct and accurate, and the physical concept is clear and the formula is simple, and is more practical.

**Key words:** Coefficient of hydrodynamic dispersion, Solute flux, Unsaturated soil.

## INTRODUCTION

With the development of society, the unreasonable discharge of industry and domestic sewage caused serious damage to the environment and caused the groundwater to be polluted to varying degrees [1]. The dispersion coefficient is the most important parameter to study the solute transport of groundwater, and the dispersion test is one of the most reliable methods to reveal the transport mechanism of solute in groundwater and to obtain the dispersion coefficient. It is indispensable for the study of groundwater pollutant transport link [2]. Dispersion test is divided into indoor tests and outdoor tests, although the outdoor test results is more in line with reality, but the actual field operation is affected by a variety of factors, and reasonable dispersion coefficient is difficult to obtain. Therefore, the laboratory tests have an important role in the study on the mechanism of the dispersion test and the influence of single factors.

In 1905 Slichter reported that solutes in the soil did not migrate at the same rate. Since then, people propo sed and gradually formed the basic theory of solute transport - hydrodynamic dispersion theory. In general, the hydrodynamic dispersion is due to the thermodynamics of the particles and the mechanical mixing of the solute molecules by the fluid, that is, the diffusion of the solute in the porous medium and the convection dispersion. There are many methods to calculate the hydrodynamic dispersion coefficient in soils at home and abroad. From the initial trial algorithm, straight line diagram method, standard curve method, point-by-point method, preferred fitting method to later linear regression method, inverse function method, microbiological method, linear regression method and so on [3, 4, 5, 6, 7]. From the whole development process, the calculation method has been from the larger hand-painted method tends to a higher degree of precision computerized, micro-processing method development [8, 9, 10, 11].

At present, the understanding of structure form in coefficient of hydrodynamic dispersion is not still unified [12]. Theoretically, the coefficient of hydrodynamic dispersion $D_{sh}$ is the sum of molecular diffusion coefficient $D_s$ and the mechanical dispersion coefficient $D_h$. With the increase of the aperture characteristic or the nonuniformity coefficient, the molecular diffusion coefficient has a small increase, the diffusion is mainly affected by concentration gradient, water content, temperature and other factors in theory [13,14]. Generally, the molecular diffusion coefficient of solute in soil is only expressed for the function of soil water content, having nothing to do with the density of solute, and it usually is shown by empirical formula [15]. Kemper et al. (1966) using the empirical formula to show the molecular diffusion coefficient $D_s$ is as following.

$$D_s = D_0 a e^{b\theta} \tag{1}$$

Where Ds is the molecular diffusion coefficient (cm2/min), D0 is the diffusion coefficient of solute in the free water body, θis soil water content (cm3/cm3), a and b are the empirical constants.

According to the literature [16], while the soil water suction changes in the range of 0.3 ~ 15 atm, b =10 is more suitable in the above empirical

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formula, the changing range of value of a is 0.005 ~ 10.001 (silt loam to clay), the bigger the soil viscidity is, the smaller the value of a.

It is generally acknowledged, in one-dimensional flow, the mechanical dispersion coefficient Dh is in proportion to the one power of average flow velocity in soil voids [17].

$$D_h = \alpha |v| \tag{2}$$

Where Dh is the mechanical dispersion coefficient (cm2/min), v is the average porous flow velocity (cm/min), α is the dispersion degree (an empirical constant) (cm).

In the above foregoing, the dispersion coefficient Dsh stands for the sum of the molecular diffusion coefficient Ds and the mechanical dispersion coefficient Dh, that is,

$$D_{sh} = D_0 a e^{b\theta} + \alpha |v| \tag{3}$$

When the speed of convection is quite large, the function of the mechanical dispersion will greatly exceed the molecular diffusion, so only to need to consider the mechanical dispersion function in hydrodynamic dispersion. On the contrary, when the soil solution is static, the mechanical dispersion is completely inoperative, only leaving the molecular spreading. Generally speaking, there are molecular dispersion function and mechanical dispersion function at the same time in solute transfer in soil, but it is difficult to distinguish them, so the molecular diffusion and the mechanical dispersion are referred to as hydrodynamic dispersion. In practical application, some scholars express the coefficient of hydrodynamic dispersion as the exponential function of the coefficient of molecular diffusion form. Smiles et al. believed that, in further theoretical and experimental work, the dispersion coefficient and velocity are independent of solute transport during the process of infiltration into very coarse soils [18]; Xie et al. systematically discussed the zero flux surface method, surface flux method, and proposed the positioning flux method [19]. They think longitude diffusion coefficient is not sensitive to the average flow velocity in soil voids, so Dsh can be alone treated as the function of the water content. However, from the literatures, many scholars express the coefficient of hydrodynamic dispersion as the linear function of the coefficient of mechanical dispersion form. They think that Dsh is in proportion to the one power of the average porous flow velocity. Zhang et al. obtained the calculation method of hydrodynamic dispersion parameters of adsorbed solute by experimental study on saturated and unsaturated hydrodynamic dispersion of light loam and heavy loam through indoor soil column [20]. Shang et al. used the normal distribution function method to obtain the hydrodynamic dispersion coefficient of the disturbed soil, and the diffusion coefficient was obtained by the proportional relationship between the hydrodynamic dispersion coefficient and the average pore velocity [21]. Li et al. carried out one-dimensional soil column of hydrodynamic dispersion and diffusion experiment based on the theory of hydrodynamics dispersion, and calculate the mechanical dispersion and diffusion coefficient of soil samples according to the experimental results [22]; Li et al. identified the dispersion coefficient of one-dimensional flow and two-dimensional hydrodynamic dispersion model based on the proportional relationship between Dsh and mean pore velocity v by the line method of the bat algorithm, and proved the practicability of the method [23]. The coefficient of hydrodynamic dispersion is expressed as in the latter form in the paper [24]. The experimental method is the indoors upright soil column.

## EXPERIMENTAL MATERIAL AND METHOD

The experimental soil was sampled from the estuary of the Changjiang River (Fig.1). The soil is silt loam, and the primitive salt content is 0.364g/kg, which is desalting soil. The soil sample used in the experiment through the natural air-drying is passed sieve mesh and mixed to well distributed. According to the actual dry density of the soil from the farmland, the soil column is filled. The degree of mineralization of supplying water in the experiment is 8g/L (close to the degree of mineralization of groundwater).

The soil column used is an upright soil column, which is made of the transparent plexiglas column, height 170cm, diameter 12.7cm. The side of the soil column is equipped with the salt sensor in order to monitor the soil salt content, and buried depth is 5, 15, 30, 45, 65, 105, 145cm, and tensiometer in the same depth to monitor the soil moisture content. Seal the bottom, open the top to evaporate. The experiment is done indoors. In order to be close to the natural environment condition, an infrared lamp is laid over the top of the column to imitate the sunlight. The bottom of soil column is equipped with Marriott bottle to control the water level and to monitor the consumption mass of water. The experiment lasted from August to October 1999, every 3 days once in the earlier and every 4 days once in the later got the observation record of soil electric conductivity, tension and the consumption mass of water. The experimental device is in Fig.2.

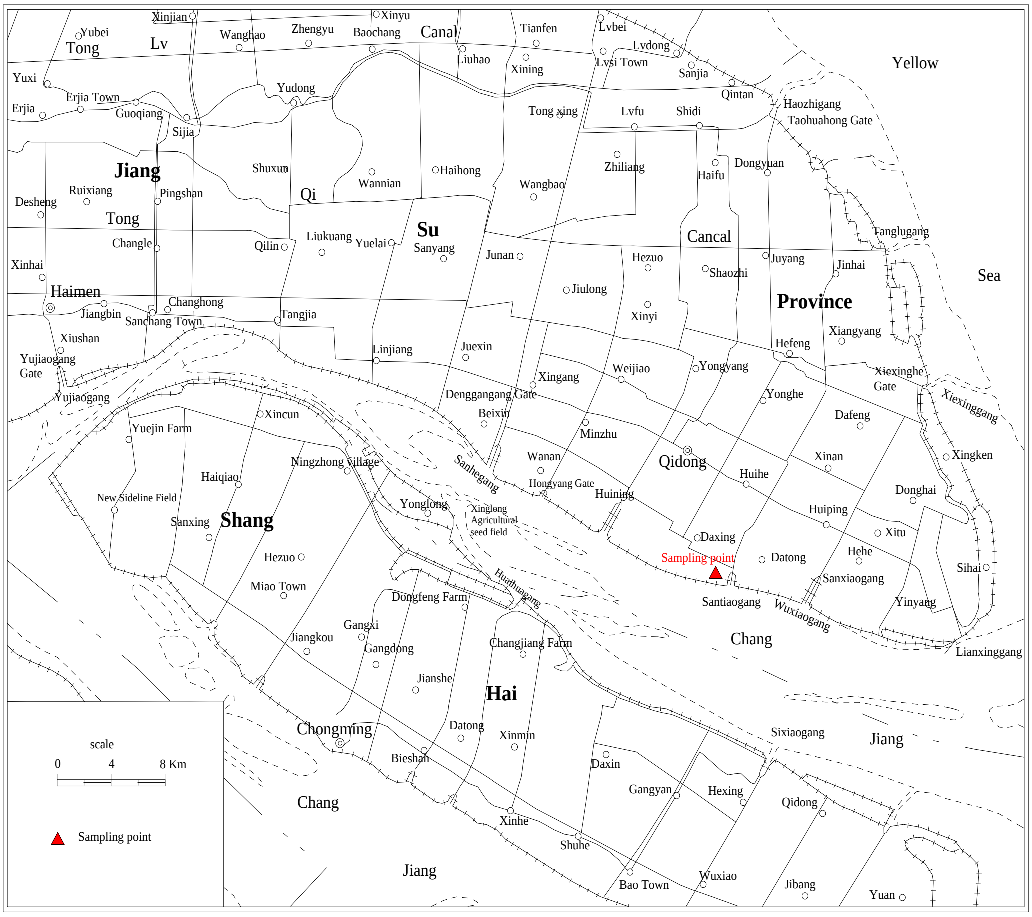

**Fig. 1.** Distribution of sampling points

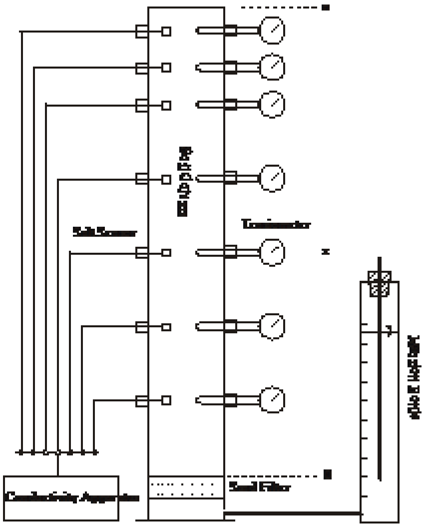

**Fig.2.** Schematic diagram showing experimental device

## CALCULATION OF COEFFICIENT OF HYDRODYNAMIC DISPERSION

To calculate the coefficient of hydrodynamic dispersion, firstly calculate the moisture flux, and then calculate the salt flux, finally determine the coefficient of hydrodynamic dispersion by the moisture flux and the salt flux.

### *Moisture flux*

If both the mass of water that the solution supplied to the soil column through the bottom and the distribution of the profile water content at different moments were known, by water mass balance principle, the moisture flux $q_z$ in any section $z$ of the soil column can be expressed as follows.

$$q_0 - q_z = \frac{1}{\Delta t}[\int_0^z \theta(t_2)dz - \int_0^z \theta(t_1)dz] \qquad (4)$$

That is

$$q_z = q_0 - \frac{1}{\Delta t}[\int_0^z \theta(t_2)dz - \int_0^z \theta(t_1)dz] \qquad (5)$$

Where qz is the moisture flux in any section of depth z (cm/d), q0 is the mass of inflow water mass from the bottom of the soil column (cm/d), θ is the volumetric water content (cm3/cm3), $\Delta t$ =t2－t1 is the time interval (d)。

The formula (5) is written in discrete form:

$$q_z^{k+1/2} = q_0^{k+1/2} - \frac{1}{\Delta t}\left[\sum_{i=0}^{i=z}\theta_{i+1/2}^{k+1}\Delta z - \sum_{i=0}^{i=z}\theta_{i+1/2}^{k}\Delta z\right] = \frac{1}{2}(q_0^{k+1} + q_0^{k}) - \frac{\Delta z}{2\Delta t}\left[\sum_{i=0}^{i=z}(\theta_{i+1}^{k+1} + \theta_{i}^{k+1}) - \sum_{i=0}^{i=z}(\theta_{i+1}^{k} + \theta_{i}^{k})\right] \tag{6}$$

Where k is the number of time intervals.

### *Salt flux*

If both the solute flux from the bottom of the soil column and the distributions of the sectional water content and of the solute density at different moments were known, by the mass conservation law, the solute flux $J_z$ in any section depth of z can be got:

$$J_0 - J_z = \frac{1}{\Delta t}[\int_0^z \theta(t_2)c(t_2)dz - \int_0^z \theta(t_1)c(t_1)dz] \tag{7}$$

That is,

$$J_z = J_0 - \frac{1}{\Delta t}[\int_0^z \theta(t_2)c(t_2)dz - \int_0^z \theta(t_1)c(t_1)dz] \tag{8}$$

Where Jz is the solute flux in any section depth of z (g/cm$^2$⁀d), J0 is the solute flux in the bottom of the soil column (g/cm$^2$⁀d), c is the density of soil solution (g/cm3), θis the volumetric water content (cm3/cm3), $\Delta t$ =t2—t1 is the time interval (d).

The equation (8) is written in dispersed form:

$$J_z^{k+1/2} = J_0^{k+1/2} - \frac{1}{\Delta t}\left[\sum_{i=0}^{i=z}\theta_{i+1/2}^{k+1}c_{i+1/2}^{k+1}\Delta z - \sum_{i=0}^{i=z}\theta_{i+1/2}^{k}c_{i+1/2}^{k}\Delta z\right] = \frac{1}{2}(J_0^{k+1} + J_0^{k}) - \frac{\Delta z}{4\Delta t}\left[\sum_{i=0}^{i=z}(\theta_{i+1}^{k+1} + \theta_{i}^{k+1})(c_{i+1}^{k+1} + c_{i}^{k+1}) - \sum_{i=0}^{i=z}(\theta_{i+1}^{k} + \theta_{i}^{k})(c_{i+1}^{k} + c_{i}^{k})\right] \tag{9}$$

### *Coefficient of Hydrodynamic Dispersion*

According to the hydrodynamic dispersion principle, solute flux is equal to the sum of the hydrodynamic dispersion flux and the convection flux, that is:

$$J = -\theta D_{sh}\frac{\partial c}{\partial z} + qc \tag{10}$$

$$D_{sh} = \frac{1}{\theta\frac{\partial c}{\partial z}}(qc - J) \tag{11}$$

Where J is the solute flux (g/cm$^2$⁀d), Dsh is the coefficient of hydrodynamic dispersion (cm2/d), c is the density of solute (g/cm3), θ is the volumetric water content, $\Delta t$ =t2-t1 is the time interval (d).

The equation (11) is written in dispersed form:

$$(D_{sh})_z^{k+1/2} = \frac{1}{\theta_z^{k+1/2}\frac{\Delta c}{\Delta z}\Big|_z^{k+1/2}}[(qc)_z^{k+1/2} - J_z^{k+1/2}] = \frac{1}{\frac{1}{2}(\theta_z^{k+1} + \theta_z^{k})\frac{c_z^{k+1} - c_z^{k}}{2\Delta z}}[\frac{1}{2}q_z^{k+1/2}(c_z^{k+1} + c_z^{k}) - J_z^{k+1/2}] = \frac{2\Delta z}{(\theta_z^{k+1} + \theta_z^{k})(c_z^{k+1} - c_z^{k})}[q_z^{k+1/2}(c_z^{k+1} + c_z^{k}) - J_z^{k+1/2}] \tag{12}$$

Replacing the foregoing $q_z^{k+1/2}$ and $J_z^{k+1/2}$ into equation (12) can calculate the coefficient of hydrodynamic dispersion Dsh. If taking a series of section z, a series of Dsh can be calculated out Furthermore, Dsh and the corresponding average pore flow velocity v in soil voids can be fitted into empirical formula.

## RESULTS AND ANALYSIS

The experimental data of calculating the coefficient of hydrodynamic dispersion is shown in the following table (table 1).

Based on the experimental data of the above table, the coefficient of hydrodynamic dispersion can be calculated with the equation (12), in which the water content is calculated according to Van Genuchten model:

$$\theta = \frac{\theta_s - \theta_r}{[1 + |\alpha h|^n]^m} + \theta_r \tag{13}$$

Where, the parameters according to the actually measured data for the water characteristic curve: θs is 0.432, θr is 0.01, $\alpha$ is 0.006038, n is 1.9087501, m is 0.476097.

The conversion relation fitted by the experimental data between the electric conductivity and the salt content of soil is:

$$s = 2.888E_c + 0.1016 \qquad (14)$$

Where s is the salt content of the soil (the salt mass of one unit mass dry soil (g/kg)), Ec is electric conductivity (mS/cm).

**Table 1.** Data of hydrodynamic dispersion experiment

| Time | Water volume | Tension *h* ($cmH_2O$) | | | | | | | Electric conductivity *EC* (dS/m) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| m-d | mL | 5 | 15 | 30 | 45 | 65 | 105 | $145^*$ | 5 | 15 | 30 | 45 | 65 | 105 | $145^*$ |
| 8-26 | 108 | 153. | 137 | 135 | 107 | 82 | 51 | 0 | 1.31 | 1.109 | 2.094 | 2.947 | 3.744 | 9.438 | 14.091 |
| 8-31 | 85 | 157 | 143 | 133 | 102 | 77 | 41 | 0 | 1.26 | 1.143 | 2.172 | 3.084 | 3.995 | 9.714 | 14.362 |
| 9-4 | 87 | 150 | 137 | 133 | 107 | 77 | 51 | 0 | 1.27 | 1.139 | 2.216 | 3.205 | 4.188 | 9.928 | 14.463 |
| 9-8 | 75 | 149 | 129 | 127 | 97 | 77 | 41 | 0 | 1.29 | 1.172 | 2.278 | 3.338 | 4.374 | 10.112 | 14.632 |
| 9-25 | 123 | 149 | 126 | 122 | 92 | 69 | 26 | 0 | 1.31 | 1.261 | 2.592 | 3.994 | 5.321 | 10.908 | 14.868 |
| 9-29 | 20 | 143 | 127 | 124 | 92 | 71 | 31 | 0 | 1.33 | 1.290 | 2.667 | 4.136 | 5.490 | 11.030 | 15.003 |
| 10-25 | 33 | 143 | 129 | 124 | 102 | 77 | 41 | 0 | 1.47 | 1.342 | 2.822 | 4.481 | 6.044 | 11.367 | 15.105 |
| 10-29 | 38 | 145 | 133 | 129 | 102 | 77 | 41 | 0 | 1.47 | 1.379 | 2.890 | 4.576 | 6.137 | 11.429 | 15.172 |

* the depth（cm）

By calculating according to the experimental data, we has got the coefficient of hydrodynamic dispersion of the unsaturated slit loam.The coefficient value is in proportion to the average flow velocity in soil voids. The fitted curve is shown in Fig.3, and the correlate coefficient R=0.85, that is,

$$D_{sh} = 0.35v \qquad (15)$$

Where $D_{sh}$ is the coefficient of hydrodynamic dispersion ($cm^2$/min), v is the average flow velocity in soil voids (cm/min).

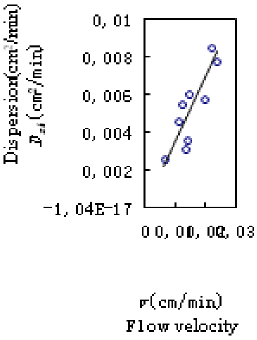

**Fig. 3.** Dsh ~ v curve fitted

In the test, as sensor and moisture tensiometer are arranged in large interval, it may influence the accuracy of the calculated result in some extent. Because solute transfer law is affected by a lot of factors, such as adsorption and desorption, the physics and chemical reaction, the original salt in soil, and so on, it is much more complicated than moisture movement. From the data of fitting figure, it can be found out that the fitting points are quite scattered, but the total linear tendency is still very clear. And the calculated values of the coefficient of hydrodynamic dispersion are quite close to the results calculated of other author, such as Yang Jinzhong, Huang Kangle, Zhang Yufang et. al. This approach is of clear physics concept and the calculated formula is simple and practical.

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