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Mathematical model of a plate heat exchanger for condensation of steam in the presence of non-condensing gas

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The process of vapour condensation from its mixture with noncondensing gas is analysed and mathematical model for condensation in PHE channels is proposed. The model is developed with accounting for the variation of local parameters of heat and mass transfer processes along condensation surface and features of these processes intensification in PHEs channels. The model is accounting for the effects of plate corrugations geometry on process intensity. The system of ordinary differential equations with considerably nonlinear right parts is solved by the numerical method of finite differences. The solution is implemented with the software developed for personal computer. The model validation is performed by comparison with experimental data for condensation of steam from its mixture with air in a sample of PHE channel.

Keywords: Plate heat exchanger, condensation, noncondensing gas, heat and mass transfer, mathematical model, experiment.

INTRODUCTION

Big amounts of heat energy used in different industries are lost with streams outgoing to the environment. Considerable quantities of such wasted heat in a form of latent heat of different condensable vapours contained in a mixture of gases are leaving out from the processes of burning fuels, of drying different materials and other industrial processes [1]. The processes like volatile organic components recovery or condensation of ammonia from synthesis gas undergone catalytic conversion also contain heat wasted with cooling streams. The efficient recovery of the heat energy in such cases requires the use of heat exchangers capable to perform the condensation process effectively and in an economically viable way. It is possible with a plate heat exchanger (PHE), which is the modern efficient type of compact heat exchangers [2]. The PHE consists from a pack of corrugated plates, as shown in Fig. 1. The plates are stamped from a thin metal sheet and being assembled in the PHE are forming robust channels of complex geometry. The multiple contact points between the adjacent plates enable sufficient constructional strength of the PHE and its ability to withstand big pressure differences between heat exchanging streams. The considerable intensification of heat and mass transfer in such channels allows substantial reduction of heat transfer area, size and weight for the same processes as compared to traditional types of tubular heat exchangers.

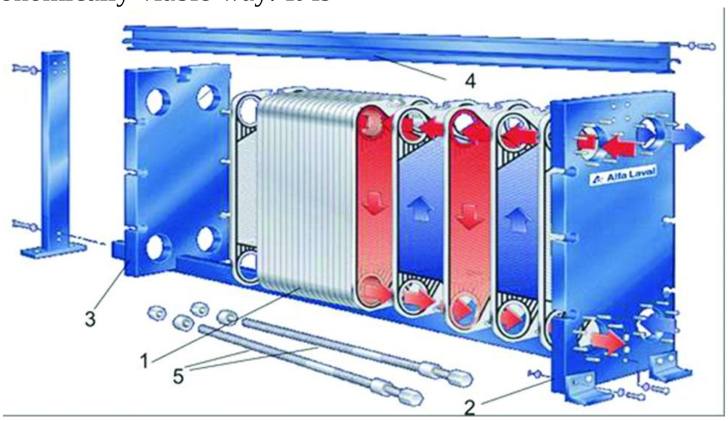

Fig. 1. The example of PHE drawing: 1 – heat transfer plate; 2 – fixed frame plate; 3 – moving frame plate; 4 – carrying bar; 5 – tightening bolts.

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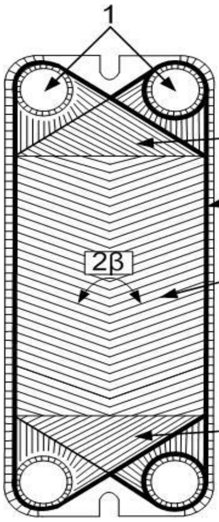

Fig. 2. Schematic drawing of a PHE plate: 1 – heat carriers inlet and outlet; 2,5 – zones for flow distribution; 3 – rubber gasket; 4 –main corrugated field.

For efficient and economically viable use of PHEs in the processes of both sensible and latent heat recovery from vapour - gas mixtures accurate enough methods for their design are required. Compared to heat transfer in single-phase conditions the condensation of vapour from its mixture with noncondensing gas is considerably complicated by the resistance to mass transfer of the condensing substance to the surface of plates, where condensed liquid film additionally creates thermal resistance for heat transfer to the cooling stream flowing in adjacent channels. The recent review of literature concerning the processes of heat and mass transfer in shell and tube condensers for condensation of vapour from its mixture with noncondensing gas is published by Huang *et al.* [3]. The substantial variation of all process parameters along the length of the heat exchanger channels necessitates utilization of mathematical models and software for adequate description of the condensing heat exchanger [4]. It requires results of experimental research of the process local parameters change in channels especially for the surfaces with enhanced heat and mass transfer [5]. In the channels of PHEs the hydrodynamics and heat and mass transfer are even more complicate because of intricate fluid flow distribution in the channels of complicated geometry. Compared to smooth tubes the effect of pressure drop on condensation in narrow channels is more important [6]. For PHEs, due to substantial differences in plate corrugations geometry, a number of different empirical correlations were developed which are adequate only for specific commercial plates in the limited experimental ranges of conditions [7].

The accurate modelling of vapour condensation in channels of PHEs must include accounting for local parameters variation along the channel length and use of reliable and accurate enough correlations for hydraulic resistance and heat and mass transfer [8]. Such mathematical model is presented in this paper with its validation according to experimental data for condensation of steam from its mixture with air in an experimental sample of PHE channel.

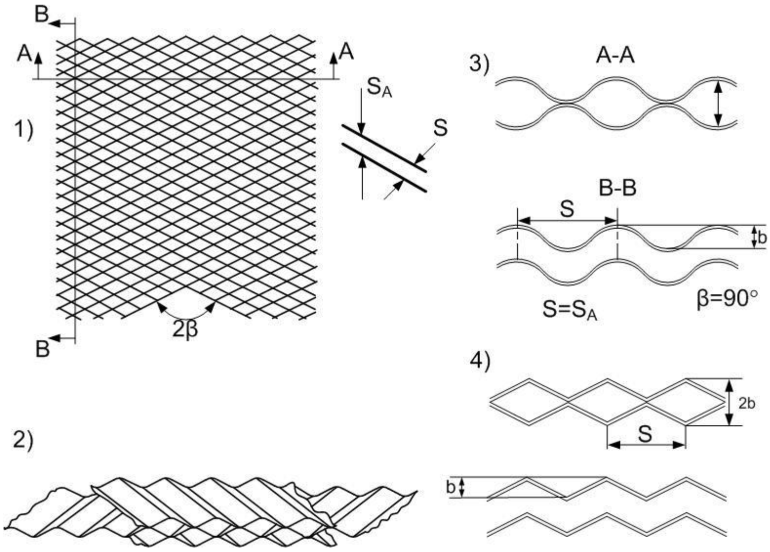

Fig. 3. Different corrugation forms: 1, 2 –intersection of the adjacent plates; 3 – channel cross-sections for the sinusoidal form of corrugations; 4 – channel cross-sections for the triangular form of corrugations.

MATHEMATICAL MODEL DEVELOPMENT

The development of a mathematical model for condensation of vapour from mixture with non-condensable gases in channels of PHE is based on the assumptions:

1. The parameters of hydrodynamic and heat –mass transfer processes on increments of the

channel are linked by the same relations as for any length of such channel with the same corrugation pattern.

2. For phase change in flow at the bulk of condensing stream equilibrium conditions of vapour are maintained and change of gaseous mixture temperature is happening due to convective heat transfer, which is also responsible for condensation in the flow core.

3. The film condensation occurs on the heat transfer surface of plates.

4. The ideal gas law is assumed for the gas-vapour mixture.

5. One-pass PHE is considered.

6. The process conditions at all PHE channels are changing in the same way and one channel surrounded by two adjacent channels with cooling media can be considered.

7. Heat losses are negligible.

The heat and material balances for the process of vapour condensation in the presence of noncondensing gas with counter-current flow of streams are written in the form of the following system of ordinary differential equations:

$$\frac{dG_v}{dx} = -\Pi \cdot j_v \tag{1}$$

$$\frac{dG_L}{dx} = \Pi \cdot j_v \tag{2}$$

$$\frac{dt_{cl}}{dx} = -\Pi \cdot q \cdot c_{cl}^{-1} \cdot G_{cl}^{-1} \tag{3}$$

$$\begin{aligned} &\frac{dt_L}{dx} \cdot c_L \cdot G_L + \\ &+\frac{dt_{mx}}{dx} \cdot c_{mx} \cdot \left(G_g + G_v\right) + \\ &+\Pi \cdot j_v \cdot r_v = \Pi \cdot q \end{aligned} \tag{4}$$

$$\begin{aligned} &\frac{dt_{mx}}{dx} \cdot c_{mx} \cdot \left(G_g + G_v\right) = \\ &= \Pi \cdot h_{cv} \cdot \left(t_{mx} - t_f\right) \end{aligned} \tag{5}$$

$$\begin{aligned} &\frac{dP_{mx}}{dx} = \frac{1}{d_e} \cdot \varsigma \cdot \frac{\rho_{mx} \cdot W_{mx}^2}{2} \cdot (1 + 2.9 \cdot X_{tt}^{0.46}) - \\ &-\frac{d}{dx}\left(\frac{\rho_{mx} \cdot W_{mx}^2}{2}\right) - \frac{d}{dx}\left(\frac{\rho_{mx} \cdot g \cdot x}{2}\right) \end{aligned} \tag{6}$$

where G_v, G_g, G_L and G_{cl} are mass flow rates of vapour, gas, condensed liquid and cooling megia in one channel, respectively, kg/s; t_{cl}, t_L, t_{mx} and t_f are cooling water, condensate, gaseous mixture and condensate film surface local temperatures, respectively, ºC; P_{mx} is the pressure of gaseous mixture, Pa; q is heat flux through the unit of plates surface, W/m^2; j_v is mass flux of vapour to the direction of heat transfer surface, kg/m^2; r_v is latent heat of vaporization, J/kg; h_{cv} is convective heat transfer film coefficient in gaseous phase, W/(m^{2}K); ρ_{mx} is density of gaseous mixture, kg/m^3; c_{cl}, c_L and c_{mx} are heat capacities of cooling media, liquid phase and gaseous mixture, respectively, J/(kg K); W_{mx} is flow velocity of gaseous mixture, m/s; Π is channel perimeter, m; x is longitudinal coordinate, m; d_e is channel equivalent diameter, m; X_{tt} is Lockhart-Martinelly parameter calculated by equation:

$$X_{tt} = \sqrt{\frac{dP_L}{dP_G}} \tag{7}$$

Here dP_L and dP_G are increments of pressure drops for liquid and gaseous phases flowing alone in a whole channel, calculated by turbulent single phase flow correlations. The system variables are also linked by algebraic equations. The relations between the saturation pressure Psat=Psat(t) and temperature $t_{sat}=t_{sat}(P)$ are determined according to data available in literature.

The temperature of the liquid film outer surface is determined by equation:

$$t_f = t_{cl} + q \cdot \left(\frac{1}{h_{cl}} + R_f + \frac{\delta_{wl}}{\lambda_{wl}} + \frac{1}{h_L}\right), \tag{8}$$

where δ_{wl} is the thickness of plate metal, m; λ_{wl} is its thermal conductivity, W/(m K); R_f is thermal resistance of fouling, (m^2 K)/W; h_L is film heat transfer coefficient from condensate film to the channel surface, W/(m^{2}K). h_L is calculated according to the equation from paper [9]:

$$h_L = \frac{\lambda_L}{d_e} Nu^* \cdot \left[1 + x_{tp} \cdot \left(\frac{\rho_L}{\rho_{mx}}\right)\right]^{0.48}. \tag{9}$$

Here λ_L is thermal conductivity of condensate, W/(m K); ρ_L is condensate density, kg/m^3; x_{tp} is mass vapour quality; Nu^* is Nusselt number calculated for liquid phase with flow rate total for both phases.

The vapour partial pressure at film surface is calculated as saturation pressure:

$$P_{vf} = Psat(t_f) \tag{10}$$

The vapour mass fraction at liquid film outer surface is:

$$y_{vf} = \left(\frac{P_{mx} - P_{vf}}{P_{vf}} \cdot \frac{M_g}{M_v} + 1 \right)^{-1} \tag{11}$$

where M_v and M_g are molar masses of non-condensable gas and vapour, respectively, kg/kmol, (for water vapour M_v = 18.015 kg/kmol, for air M_g =28.96 kg/kmol).

The mass fraction of vapour at the flow core is:

$$y_{vb} = \frac{G_v}{G_v + G_g} \tag{12}$$

The vapour partial pressure at the flow core is:

$$P_{vb} = \frac{P_{mx}}{\frac{M_v}{M_g} \cdot \frac{G_g}{G_v} + 1}, \tag{13}$$

where G_g is mass flow rate of noncondensing gas, kg/s.

The saturation temperature at the flow core is:

$$t_{satb} = tsat(P_{vb}) \tag{14}$$

and with solution of equation (5) the following condition must be satisfied:

$$t_{mx} \geq t_{satb} \tag{15}$$

The mass flux to the condensation surface is:

$$j_v = \beta_D \cdot \left(y_{vb} \cdot \rho_{mx} - y_{vf} \cdot \rho_{mxf} \right) \tag{16}$$

The coefficients of heat transfer h_{cv} and mass transfer β_D are calculated according to heat and mass transfer analogy corrected for the influence of transverse mass flux [10]:

$$h_{cv} = \left(\lambda_{mx} / d_e \right) \cdot \Psi_H \cdot Nu_0 \tag{17}$$

$$\beta_D = \left(D_D / d_e \right) \cdot \Psi_D \cdot Nu_{D0}, \tag{18}$$

where λ_{mx} is the thermal conductivity of the gaseous mixture, W/(m K); D_D is diffusivity coefficient, m^2/s.

The relative factors of heat and mass transfer are:

$$\Psi_{H(D)} = 4 \cdot \left(1 + 0.85 \cdot b_{H(D)} \right) \cdot \left(1 + \sqrt{\frac{\rho_{mx}}{\rho_{mxf}}} \right)^{-2} \tag{19}$$

Here b_H and b_D are heat and diffusivity parameters:

$$b_H = \frac{c_{Pv}}{c_{Pmx}} \cdot \frac{j_v \cdot \mathrm{Re}_{mx} \cdot \mathrm{Pr}_{mx}}{\rho_{mx} \cdot W_{mx} \cdot Nu_0}$$
$$b_D = \frac{j_v \cdot \mathrm{Re}_{mx} \cdot \mathrm{Pr}_D}{\rho_{mx} \cdot W_{mx} \cdot Nu_{D0}} \tag{20}$$

The Nusselt numbers in a single phase flow are determined by correlation for the main corrugated field of the channels of PHEs presented in [11]:

$$Nu_{0(D)} = 0.065 \cdot \mathrm{Re}^{6/7} \cdot \left(\psi \cdot \zeta / F_x \right)^{3/7} \cdot \mathrm{Pr}_{(D)}^{0.4} \tag{21}$$

$$\psi = \left(\mathrm{Re}/A \right)^{-0.15 \cdot \sin(\beta)} \text{ at } Re>A;\ \psi=1$$
at $Re \leq A$ where $A = 380 / \left[tg(\beta) \right]^{1.75}$ (22)

The friction factor for the total hydraulic resistance is calculated by the following correlation from [12]:

$$\zeta = 8 \times \left[\left(\frac{12 + p2}{\mathrm{Re}} \right)^{12} + \frac{1}{\left(A + B \right)^{\frac{3}{2}}} \right]^{\frac{1}{12}} \tag{23}$$

$$A = \left[p4 \cdot \ln \left(\frac{p5}{\left(\frac{7 \cdot p3}{\mathrm{Re}} \right)^{0.9} + 0.27 \cdot 10^{-5}} \right) \right]^{16};$$

$$B = \left(\frac{37530 \cdot p1}{\mathrm{Re}} \right)^{16},$$

where p1, p2, p3, p4, p5 are empirical parameters calculated according to plates corrugations shape:

$$p1 = \exp\left(-0.157 \cdot \beta \right);\ p2 = \frac{\pi \cdot \beta \cdot \gamma^2}{3};\ p3 = \exp\left(-\pi \cdot \frac{\beta}{180} \cdot \frac{1}{\gamma^2} \right);\ p5 = 1 + \frac{\beta}{10};$$
$$p4 = \left(0.061 + \left(0.69 + tg\left(\beta \cdot \frac{\pi}{180} \right) \right)^{-2.63} \right) \cdot \left(1 + (1 - \gamma) \cdot 0.9 \cdot \beta^{0.01} \right) \tag{24}$$

where $\gamma = 2 \cdot b/S$ – the corrugation aspect ratio; β is angle of corrugations to the main flow direction, degrees; Re is Reynolds number determined with the equivalent diameter of the channel $de=2b$; F_x is heat transfer area increase coefficient.

The equations (19)-(24) are also employed for determining heat transfer and pressure drop in the flow of cooling fluid, and in equations (6) and (9). It makes possible accounting in mathematical model for the effects of plate corrugations geometrical parameters.

The equations (1)-(24) and correlations determining temperature and pressure effects on thermal and physical properties of substances together with geometrical relations for PHE channel are representing the system of ordinary differential equations. The right parts of the system equations are nonlinear that does not permit system analytical solution. The numerical solution by the finite difference method is implemented as software for PC using Mathcad.

EXPERIMENTAL PART

To validate the developed mathematical model experiments for condensation of a steam – air mixture in a sample of PHE channel were performed. For experimental study four corrugated plates were stamped from stainless steel AISI 304. The experimental model (Fig. 4) consisted of four plates welded together to form three inter-plate channels. The channel on the gas - steam side was formed by combination of two plates with a corrugation angle $\beta = 60°$.

The saturated steam-air mixture was directed and condensed in the central channel. It was cooled by a water flow in two periphery channels having thermal insulation on the outside. The temperatures of gaseous mixture and cooling media were measured by copper-constantan thermocouples with an accuracy of ±0.1 ºC. The temperature measuring points are situated at the inlet and exit of heat-exchanging streams and at seven points along the channel. The pressure of the gaseous stream is measured by pressure gauges at the inlet and exit of the channel with an accuracy of ±0.005 bar. The mass flow rate of the cooling water is determined with the use of orifice flow meter, accuracy ±1 %. The flow rate of incoming air is measured using a set or rotameters, with minimal accuracy of ±2 %. The volumetric flow rate of the water condensate created in the channel was measured by a set of measuring vessels with an accuracy of ±1 %. The steam flow rate is determined by summing the water condensate flow rate with the flow rate of not condensed steam exiting the channel with outgoing steam-air mixture at saturation conditions. The channel model is 1 meter long and its width is 0.225 m. The corrugation height is b = 5 mm, thickness of the plate is δ = 1 mm, corrugation angle β = 60º, aspect ratio γ = 0.556 and area increase coefficient Fx = 1.15. The experiments included 48 tests at different conditions of gas-steam mixture condensation. The absolute pressure was changed in the range from 2.93 to 1.025 bar; the air volume fraction in the entering channel mixture was in the range from 2.8 % to 70 %; the local velocity of gaseous stream was in the range from 46 to 4.1 m/s; the temperature of gaseous stream changed in the range from 88.2 to 115.1 ºC; the temperature of cooling media varied from 23.8 to 71.5 ºC.

MODEL VALIDATION AND DISCUSSION OF THE RESULTS

The simulations of the condensation process parameters are performed using computer software developed according to the presented mathematical model. The calculations are made based on conditions of the tests.

There are specified mass flow rates of steam, air, cooling water, temperatures and pressures of incoming gas-vapour mixture and of incoming cooling water. The simulated results are compared to the results of the experimental runs. The differences between calculated and experimental total heat loads are not bigger than ±2.8 % for all performed experiments.

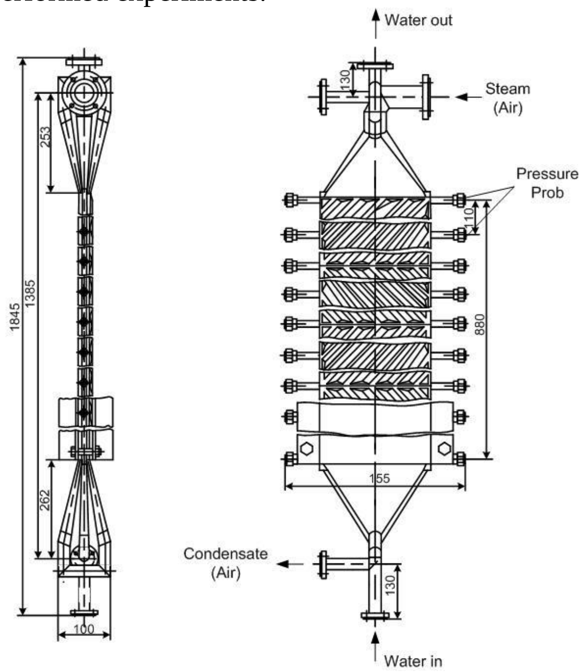

Fig. 4. Schematic drawing of the experimental sample of PHE channel.

The discrepancies in calculated and measured temperature of gas-vapour mixture at channel outlet were lower than ±2.5 ºC. Such accuracy can be regarded as acceptable for calculations of PHEs in industry for heat recovery and utilization of waste heat.

In Fig. 5 are shown the graphs of calculated local temperatures along the PHE channel for the test run with mass air content in the incoming mixture of 3%. The accuracy of predicting local temperatures of gaseous stream is in the limits of ±3.1 ºC. The biggest errors are observed closer to the end of the channel, where allmost all the steam is condensed. It makes the relative error in prediction of remaining small quantities of gaseous steam more significant, than at the initial stages of condensation.

The temperature change is most significant at the end of the channel, while the temperature drop at the initial channel sections is rather small. It demonstrates the importance of modelling with accounting for change of the local process parameters and all its characteristics along the surface of condensation.

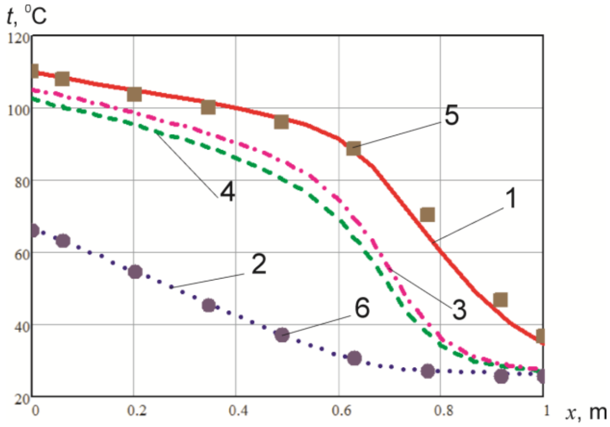

Fig. 5. Change of temperatures in a sample of PHE channel at air mass fraction of 3%. Calculated results are presented by solid curves, experimental data by dots. Calculated: 1 – gaseous stream; 2 - cooling water; 3 – liquid film; 4 - wall. Experimental: 5 – gaseous stream; 6 - cooling water.

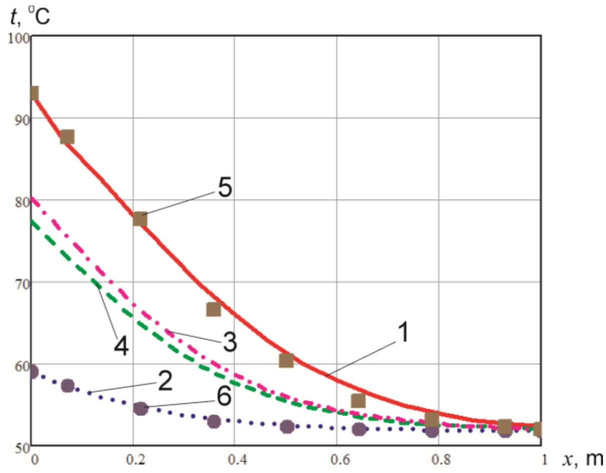

Fig. 6. Change of temperatures in a sample of PHE channel at air mass fraction of 55%.Calculated results are presented by solid curves, experimental data by dots. Calculated: 1 – gaseous stream; 2 - cooling water; 3 – liquid film; 4 - wall. Experimental: 5 – gaseous stream; 6 - cooling water.

With the increase of initial content of air in the mixture the character of process parameters distribution is changing, as it is shown in Fig. 6 for the test run with initial air mass fraction of 55%. Here the most changes in temperature are at the sections close to gaseous mixture inlet and prediction of the temperature at the channel exit is more accurate. Such cases are incountered with the use of PHEs for utilisation of waste heat from condensible gaseous streams and optimisation of their cost in cases like the case of drying process considered in paper [1], where the model presented here was used for PHE selection based on economical objectives. The accounting for the effect on heat transfer area and cost of PHE of plates corrugation parameters is also making the model a valuable tool when optimising PHEs plates geomery.

CONCLUSIONS

The accurate modelling of PHEs for utilisation of heat energy from condensing gaseous streams requires accounting for the variation of local parameters of heat and mass transfer processes along the length of condensation surface. The proposed mathematical model is based on a system of ordinary differential equations with strong nonlinearity of their right parts. For solution of this system the numerical method of finite differences is used, which is implemented on a PC. The validation and acceptable accuracy of the model is confirmed by comparison with data of experiments for condensation of steam from its mixture with air in a sample of PHE channel. The model is accounting for the effect of plate geometry on the process intensity and can be used for optimisation of PHE geometrical parameters for condensation processes, as well as for optimal selection of PHEs for waste heat utilisation from condensable gaseous streams in industry.

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МАТЕМАТИЧЕСКИ МОДЕЛ НА ПЛАСТИНЧАТ ТОПЛООБМЕННИК ЗА КОНДЕНЗАЦИЯ НА ВОДНА ПАРА В ПРИСЪСТВИЕ НА НЕКОНДЕНЗИРАЩ ГАЗ

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(Резюме)

Анализиран е процесът на кондензация на пара от смес с некондензиращ газ и е предложен математичен модел за кондензация в каналите на пластинчат топлообменник. Моделът е разработен с отчитане на промяната на локалните топлинни параметри и масопреносните процеси по протежение на кондензационната повърхност и характеристиките на интензифицирането на тези процеси в каналите на пластинчатия топлообменник. Методът отчита влиянието на геометрията на гофриране на пластините върху интензивността на процеса. Системата от обикновени диференциални уравнения със значително нелинейни десни части е решена по числовия метод на крайните разлики. Решението е осъществено със софтуер, разработен за персонален компютър. Моделът е валидиран чрез сравняване с експериментални данни за кондензация на водна пара от смес с въздух в канал на пластинчат топлообменник.